Some considerations on optimisation-based control in robotics

Many problems, some ideas towards solutions

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Interactive robots do not exist for real

Real-world ...

Basic locomotion and manipulation skills

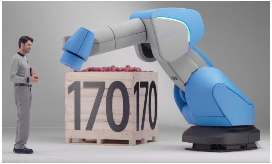
Advanced locomotion skills

Cognitive and physical interactions

Interactive robots do not exist for real

... vs Laboratory science and technology

Advanced control but no living bodies around



How many (trully) collaborative robots have you seen in the industry ?

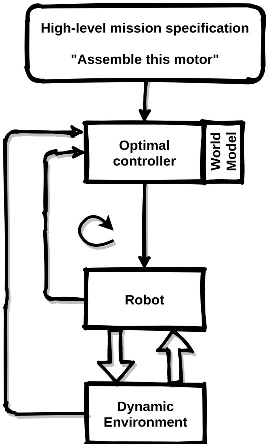
Why is it so ?

The world is dynamic, complex and hard to predict (impact in 6s)

Outline of the presentation

(Reactive) Optimal control

Ideally, solve reactively ...



$$\min_{t_0, t_f, \boldsymbol{x}(t), \boldsymbol{u}(t)} \underbrace{J_b(t_0, t_f, x(t_0), x(t_f))}_{boundary\ \ objective\ \ function} + \underbrace{\int_{t_0}^{t_f} J_i(s, x(s), u(s))ds}_{integral\ \ objective\ \ function}$$

subject to :

- Dynamics : $\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{u}(t))$
- Path constraints : $\boldsymbol{h}(t, \boldsymbol{x}(t), \boldsymbol{u}(t)) \leq \boldsymbol{0}$
- State constraints : $\boldsymbol{x}_l(t) \leq \boldsymbol{x}(t) \leq \boldsymbol{x}_u(t)$
- Control bounds : $\boldsymbol{u}_l(t) \leq \boldsymbol{u}(t) \leq \boldsymbol{u}_u(t)$

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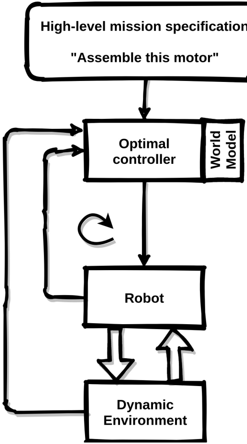

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... but in practice

- infinite dimensional problem
- can generally not be solved, even once

$\hookrightarrow$ transformed in a finite dimensional problem : non linear program / constrained parameter optimization

$\hookrightarrow$ hard to solve, cannot be solved reactively

Looking closer

In dynamic environments, $\boldsymbol{x}(t) = \{\boldsymbol{x}_{rob}(t), \boldsymbol{x}_{env}(t)\}$
$\hookrightarrow$ requires **perception** for the state of the environment $\boldsymbol{x}_{env}(t)$
$\hookrightarrow$ no control over $\boldsymbol{x}_{env}(t) \rightarrow$ reactive planning needed

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  - $\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{b}(\boldsymbol{q}, \boldsymbol{\nu}) = \boldsymbol{S}^T(\boldsymbol{q})\boldsymbol{\tau}\ (+\sum_i^{n_c} \boldsymbol{J}_{c_i}^T(\boldsymbol{q})\boldsymbol{f}_{c_i})$
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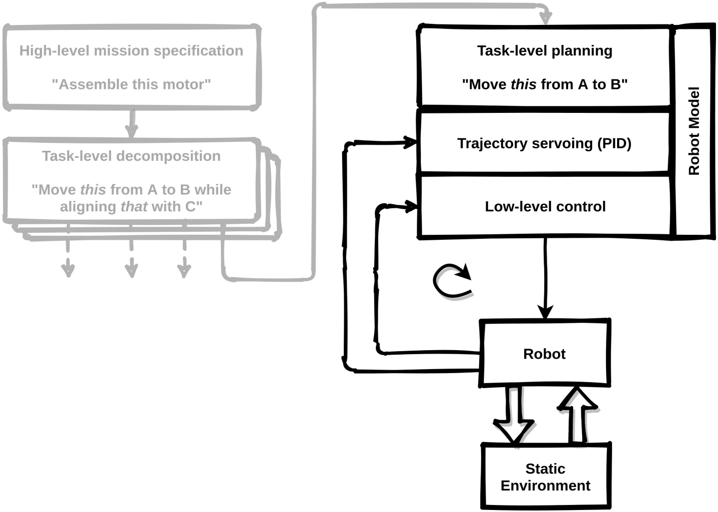
  - $\boldsymbol{v}_i = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{\nu}}\ \ \forall i \in [1, n_o]$ and $\boldsymbol{v}_i := \dot{\boldsymbol{H}}_i$
- Constraints :
  - $\boldsymbol{\tau}_l \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_u$
  - $\dot{\boldsymbol{\tau}}_l \leq \dot{\boldsymbol{\tau}} \leq \dot{\boldsymbol{\tau}}_u$
  - $\boldsymbol{q}_l \leq \boldsymbol{q} \leq \boldsymbol{q}_u$
  - $\dot{\boldsymbol{\nu}}_l \leq \dot{\boldsymbol{\nu}} \leq \dot{\boldsymbol{\nu}}_u$
  - $\boldsymbol{h}(\boldsymbol{x}_{env}, \boldsymbol{q}) \leq \boldsymbol{0}$
  - ...

$\hookrightarrow$ **very complex and computationnally demanding control / optimization problem**

Outline of the presentation

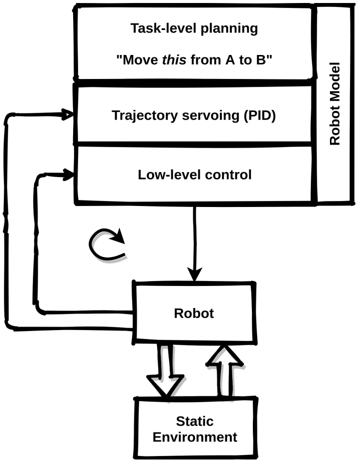
Optimal control vs real-life

Historically in the industry, the problem left to robots is simplified



Optimal control vs real-life

Static environment → reactivity not required at the task planning level ...



... as constraints are met

- offline, through planning
- a posteriori through emergency stops or stereotypical safety zones definition

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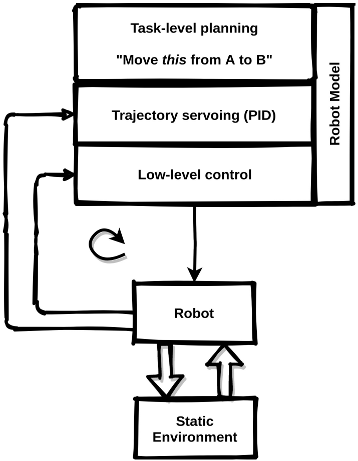

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Yet finding a control trajectory is complex

↪ Decouple planning and control

- Plan for $\boldsymbol{q}(t)$ or $\boldsymbol{H}(t)$
- Perform trajectory servoing and low level-control

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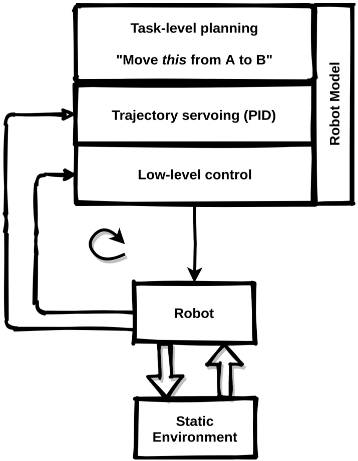

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Still too complex !

- Simplification based on an underestimation of the true robot capacities

$\hookrightarrow$ the industry is full of oversized and dangerous robots

- Highly expert manual tuning required

$\hookrightarrow$ robots are not the promised versatile tools

Illustration with the Franka Emika Panda Robot

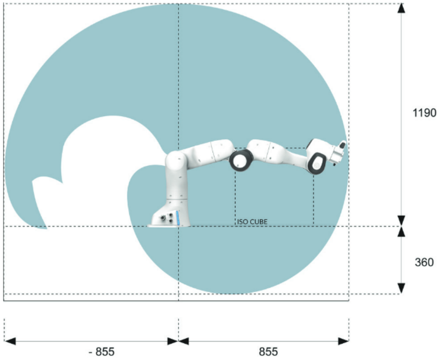
Constants

Limits in the Cartesian space are as follows:

Name	Translation	Rotation	Elbow
$\dot{p}_{max}$	1.7000 $\frac{\text{m}}{\text{s}}$	2.5000 $\frac{\text{rad}}{\text{s}}$	2.1750 $\frac{rad}{\text{s}}$
$\ddot{p}_{max}$	13.0000 $\frac{\text{m}}{\text{s}^2}$	25.0000 $\frac{\text{rad}}{\text{s}^2}$	10.0000 $\frac{rad}{\text{s}^2}$
$\dddot{p}_{max}$	6500.0000 $\frac{\text{m}}{\text{s}^3}$	12500.0000 $\frac{\text{rad}}{\text{s}^3}$	5000.0000 $\frac{rad}{\text{s}^3}$

Joint space limits are:

Name	Joint 1	Joint 2	Joint 3	Joint 4	Joint 5	Joint 6	Joint 7	Unit
q_{max}	2.8973	1.7628	2.8973	-0.0698	2.8973	3.7525	2.8973	rad
q_{min}	-2.8973	-1.7628	-2.8973	-3.0718	-2.8973	-0.0175	-2.8973	rad
$\dot{q}_{max}$	2.1750	2.1750	2.1750	2.1750	2.6100	2.6100	2.6100	$\frac{\text{rad}}{\text{s}}$
$\ddot{q}_{max}$	15	7.5	10	12.5	15	20	20	$\frac{\text{rad}}{\text{s}^2}$
$\dddot{q}_{max}$	7500	3750	5000	6250	7500	10000	10000	$\frac{\text{rad}}{\text{s}^3}$
$\tau_{j\,max}$	87	87	87	87	12	12	12	Nm
$\dot{\tau}_{j\,max}$	1000	1000	1000	1000	1000	1000	1000	$\frac{\text{Nm}}{\text{s}}$



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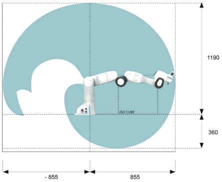
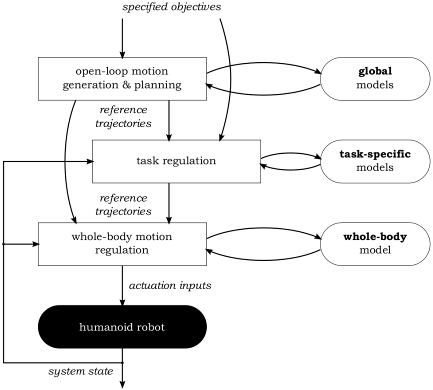


↪ **Curse of "collaborative" robotics**

- Safety in the collaboration requires small robots and controlled stops
- Small robots capabilities are small
- Underestimating the capabilities of small robots leads to "not much" capabilities
- Potentially safe robots are mostly useless

Optimal control vs complex robots (e.g. humanoids)

For systems making intermittent contacts with the environment (e.g. humanoids walking)...



[Ibanez 2017]

... mostly two solutions

- Sequential simplified planning problem solving from contact sequence to center of mass trajectory under balance constraints and in purely static environment (plan once)
- Stereotypical walking gaits (planned once) on flat grounds and online planar trajectory adaptation

\+ Trajectory servoing and multi-task whole-body control

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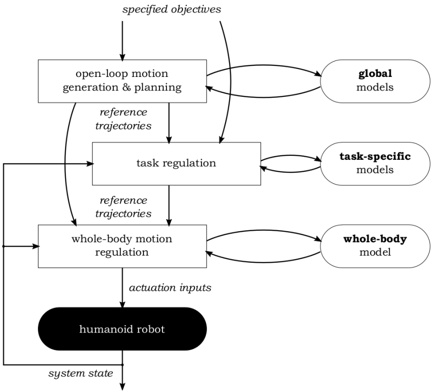

[Ibanez 2017]

Difficulties

- Planning performed with advanced models is costly $\rightarrow$ no reactivity
- Simplified models do not account for the true capabilities of the system

$\hookrightarrow$ underestimation / overstimation $\rightarrow$ manual tuning

- Humanoids can't do much in real life

Outline of the presentation

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Robot low-level control as an optimisation problem

In a dynamic environment, performance and safety requires to embed constraints in the low-level control problem : at each control instant, find the actuation torque $\boldsymbol{\tau}^*$ optimizing under constraints some objective related task $\boldsymbol{v}^* = \boldsymbol{J}(\boldsymbol{q})\boldsymbol{\nu}$

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- **Standard** IVK and operational space control **approaches***
  $\hookrightarrow$ solution based on $\boldsymbol{J}^+$ and null-space projections $\dot{\boldsymbol{\nu}} = \boldsymbol{J}^+(\boldsymbol{q})\boldsymbol{v} + (\boldsymbol{I} - \boldsymbol{J}^+\boldsymbol{J})\dot{\boldsymbol{\nu}}_0$

. *see the work of [Liégeois 1977], [Khatib 1987], [Siciliano 1991], [Chiaverini 1997], [Mansard 2009], [Flacco 2012],...

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- These constraints are linear wrt control variables : **convex solution space**
  $\hookrightarrow$ convex optimization (LQP) is a powerful tool to solve optimally the reactive control problem.

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Robot low-level control as an optimisation problem

3 reasons why Quadratic Programs are better than explicit Jacobian inversions

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3 reasons why Quadratic Programs are better than explicit Jacobian inversions

1. Leave your robot alone
   - Methods based on $\boldsymbol{J}^{+}$ forces constraints to be treated as tasks $\rightarrow$ active avoidance
   - QP allows to consider constraints as such $\rightarrow$ passive avoidance

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2. More constraints than DoFs : choose which one to consider at each time
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2. More constraints than DoFs : choose which one to consider at each time
   - Methods based on $\boldsymbol{J}^{+}$ use context specific heuristics to do so
   - QP comes with an optimal active constraints determination algorithm
3. Infeasibilty can't be ignored
   - Methods based on $\boldsymbol{J}^{+}$ can solve infeasible problems $\rightarrow$ constraints violation
   - QP can't be solved if infeasible $\rightarrow$ deal with this problem first
     [Rubrecht 2012, Meguenani 2017b, Del Prete 2018]

Constraints compliance as a control feature

For example :

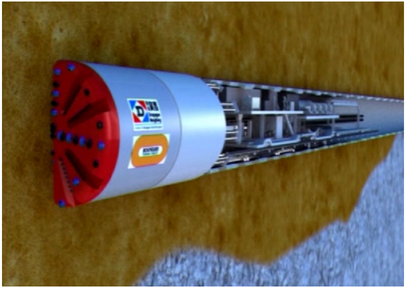
$$\boldsymbol{\tau}_{k+1}{}^{*} = \underset{\boldsymbol{\tau}_{k+1}, \ddot{\boldsymbol{q}}_{k+1}}{\arg\min} \left\| \boldsymbol{obj}\left(\ddot{\boldsymbol{q}}_{k+1}, \ddot{\boldsymbol{x}}^{*}_{k+1}\right) \right\|^2_{\boldsymbol{Q}_t} + \epsilon \left\| \begin{bmatrix} \boldsymbol{\tau}_{k+1} \\ \ddot{\boldsymbol{q}}_{k+1} \end{bmatrix} \right\|^2_{\boldsymbol{Q}_r}$$

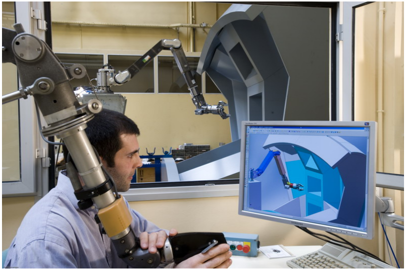
$$\begin{aligned} \text{such that} \quad \boldsymbol{M}(\boldsymbol{q}_k)\ddot{\boldsymbol{q}}_{k+1} + \boldsymbol{b}(\boldsymbol{q}_k, \dot{\boldsymbol{q}}_k) &= \boldsymbol{S}^T(\boldsymbol{q}_k)\boldsymbol{\tau}_{k+1} \\ \boldsymbol{\tau}_{min} \leq \boldsymbol{\tau}_{k+1} &\leq \boldsymbol{\tau}_{max} \\ \boldsymbol{q}_{min} \leq \boldsymbol{q}_{k+1} &\leq \boldsymbol{q}_{max} \\ \dot{\boldsymbol{q}}_{min} \leq \dot{\boldsymbol{q}}_{k+1} &\leq \dot{\boldsymbol{q}}_{max} \\ 0 \leq d^{rob,obj_j}_{k+1} & \quad \forall j \in \{1, ..., n_{obj}\} \end{aligned}$$

$$\boldsymbol{obj}\left(\ddot{\boldsymbol{q}}_{k+1}, \ddot{\boldsymbol{x}}^{*}_{k+1}\right) = \underbrace{\ddot{\boldsymbol{x}}^{des}_{k+1} + PD(\boldsymbol{x}_k, \boldsymbol{x}^{des}_{k+1})}_{\ddot{\boldsymbol{x}}^{*}_{k+1}} - \boldsymbol{J}(\boldsymbol{q}_k)\ddot{\boldsymbol{q}}_{k+1} - \dot{\boldsymbol{J}}(\boldsymbol{q}_k)\dot{\boldsymbol{q}}_k$$

Constraints compliance as a control feature : the teleoperation case

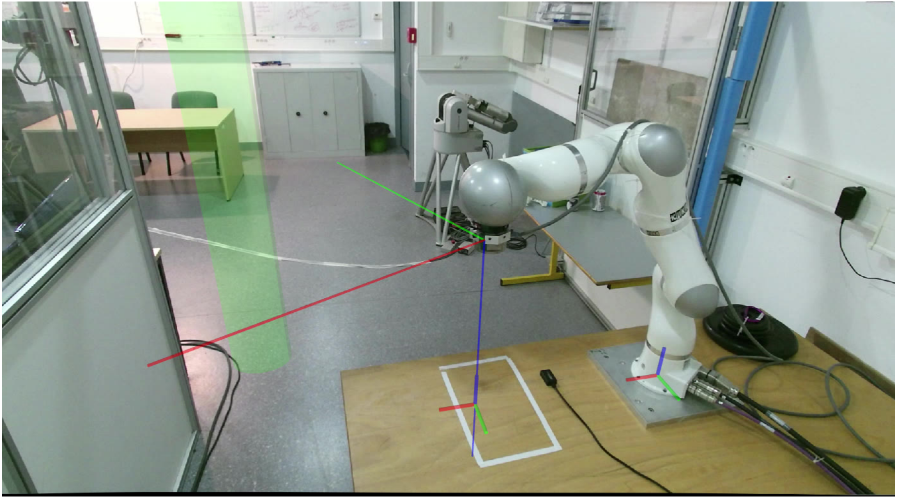
- PhD thesis Sébastien Rubrecht, ANR TELEMACH, CIFRE Bouygues Construction [Rubrecht 2010, Rubrecht 2011, Rubrecht 2012]
- Context : Teleoperation in tunnel boring machine cutter-heads
- Static environment, interactive task definition





Constraints compliance as a control feature

- PhD work of Lucas Joseph, CIFRE GE Healthcare [Joseph 2018c]
- Dynamic environment : perception in the loop and reactive constraints adaptation



Outline of the presentation

Where is redundancy hiding ?

- Classically, it's considered to be related to the null-space of the Jacobian
  $\dot{\boldsymbol{\nu}} = \boldsymbol{J}^{+}(\boldsymbol{q})\boldsymbol{v} + (\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\dot{\boldsymbol{\nu}}_0$ or $\boldsymbol{\tau} = \boldsymbol{J}^{T}(\boldsymbol{q})\boldsymbol{f} + (\boldsymbol{I} - \boldsymbol{J}^{T}\boldsymbol{J}^{T+})\boldsymbol{\tau}_0$

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- In a QP, it does not appear explicitely. Three possibilities :

1. Write the cost function as a weighted sum of individual task constraints [Salini 2011],[Bouyarmane 2011]

$$\boldsymbol{\tau}^* = \arg\min_{\mathbb{X}} \quad T(\mathbb{X}) = \sum_{i=1}^{n_o} T_i(\mathbb{X}, \boldsymbol{W}_i) + w_0 T_0 \tag{1}$$

$$\text{subject to} \quad \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{b}(\boldsymbol{q}, \boldsymbol{\nu}) = \boldsymbol{S}^{T}(\boldsymbol{q})\boldsymbol{\tau} + \sum_{i=1}^{n_c} \boldsymbol{J}_{c_i}^{\top}(\boldsymbol{q})\boldsymbol{f}_{c_i} \tag{2}$$

$$\boldsymbol{A}(\boldsymbol{q}, \boldsymbol{\nu})\mathbb{X} = \boldsymbol{b}(\boldsymbol{q}, \boldsymbol{\nu}) \tag{3}$$

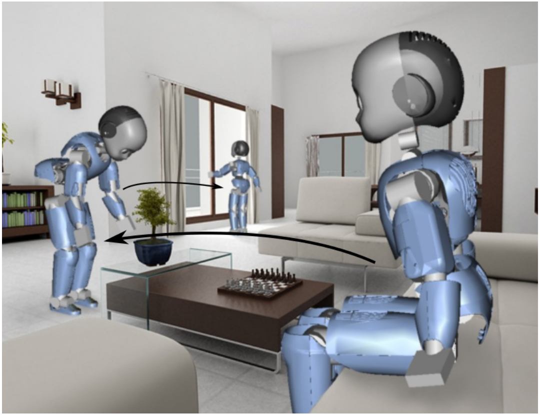
$$\boldsymbol{D}(\boldsymbol{q}, \boldsymbol{\nu})\mathbb{X} \leq \boldsymbol{h}(\boldsymbol{q}, \boldsymbol{\nu}) \tag{4}$$

Where is redundancy hiding ?

- Classically, it's considered to be related to the null-space of the Jacobian $\dot{\boldsymbol{\nu}} = \boldsymbol{J}^{+}(\boldsymbol{q})\boldsymbol{v} + (\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\dot{\boldsymbol{\nu}}_0$ or $\boldsymbol{\tau} = \boldsymbol{J}^{T}(\boldsymbol{q})\boldsymbol{f} + (\boldsymbol{I} - \boldsymbol{J}^{T}\boldsymbol{J}^{T+})\boldsymbol{\tau}_0$

- In a QP, it does not appear explicitely. Three possibilities :

1. Write the cost function as a weighted sum of individual task constraints [Salini 2011],[Bouyarmane 2011]



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2. Solve a cascade of n_o QPs to ensure a strict hierarchy [Kanoun 2009], [Escande 2014]

$$\boldsymbol{\tau}^* = \underset{\mathbb{X}}{\arg\min} \quad T_i(\mathbb{X}) \tag{1}$$

$$\text{subject to} \quad \boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{\nu}} + \boldsymbol{b}(\boldsymbol{q},\boldsymbol{\nu}) = \boldsymbol{S}^{T}(\boldsymbol{q})\boldsymbol{\tau} + \sum_{i=1}^{n_c} \boldsymbol{J}_{c_i}^{\top}(\boldsymbol{q})\boldsymbol{f}_{c_i} \tag{2}$$

$$\boldsymbol{A}(\boldsymbol{q},\boldsymbol{\nu})\mathbb{X} = \boldsymbol{b}(\boldsymbol{q},\boldsymbol{\nu}) \tag{3}$$

$$\boldsymbol{D}(\boldsymbol{q},\boldsymbol{\nu})\mathbb{X} \leq \boldsymbol{h}(\boldsymbol{q},\boldsymbol{\nu}) \tag{4}$$

$$T_j(\mathbb{X}) = T_j^* \quad \forall j < i \tag{5}$$

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1. Write the cost function as a weighted sum of individual task constraints [Salini 2011],[Bouyarmane 2011]

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3. Solve a QP allowing the formulation and the smooth transition between both soft and strict hierarchy – Generalized Hierarchical Control [Liu 2016]

$$\boldsymbol{\tau}^* = \underset{\boldsymbol{\tau}, \boldsymbol{f}_c, \dot{\boldsymbol{\nu}}'}{\arg\min} \quad T(\mathbb{X}) = \sum_{i=1}^{n_o} T_i(\boldsymbol{\tau}, \boldsymbol{f}_c, \dot{\boldsymbol{\nu}}'_i) \tag{1}$$

$$\text{subject to} \quad \boldsymbol{M}(\boldsymbol{q})\boldsymbol{P}\dot{\boldsymbol{\nu}}' + \boldsymbol{b}(\boldsymbol{q}, \boldsymbol{\nu}) = \boldsymbol{S}^{T}(\boldsymbol{q})\boldsymbol{\tau} + \sum_{i=1}^{n_c} \boldsymbol{J}_{c_i}^{\top}(\boldsymbol{q})\boldsymbol{f}_{c_i} \tag{2}$$

$$\boldsymbol{A}(\boldsymbol{q}, \boldsymbol{\nu})[\boldsymbol{\tau}^{T}, \boldsymbol{f}_c^{T}, \boldsymbol{P}\dot{\boldsymbol{\nu}}'^{T}]^{T} = \boldsymbol{b}(\boldsymbol{q}, \boldsymbol{\nu}) \tag{3}$$

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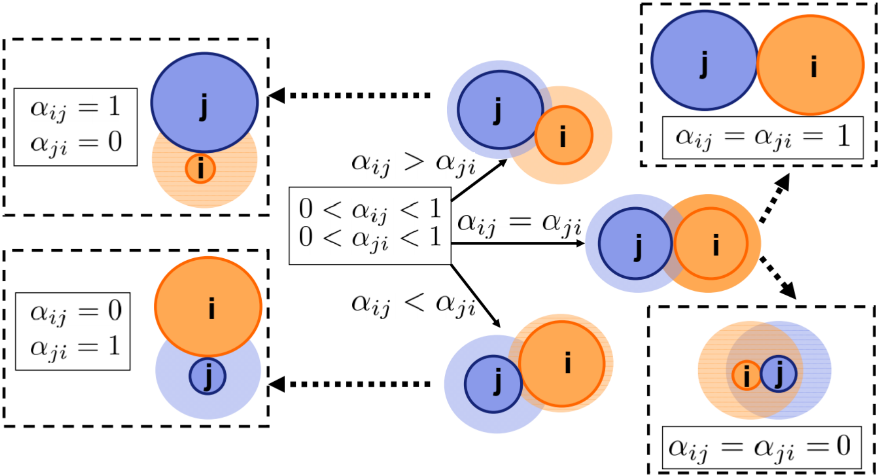
Priorities in Generalized Hierarchical Control [Liu 2016]



- Task activation $\alpha_{ii} = 1 \longrightarrow \alpha_{ii} = 0$
- Task deactivation $\alpha_{ii} = 0 \longrightarrow \alpha_{ii} = 1$

Redundancy as a key to simple adaptive behaviours

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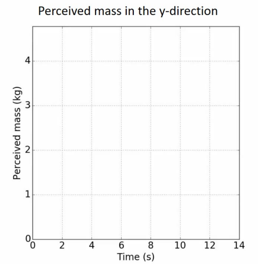

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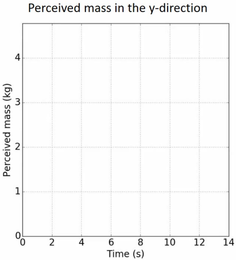
- Apparent mass minimization in the potential direction of interaction [Joseph 2018a]



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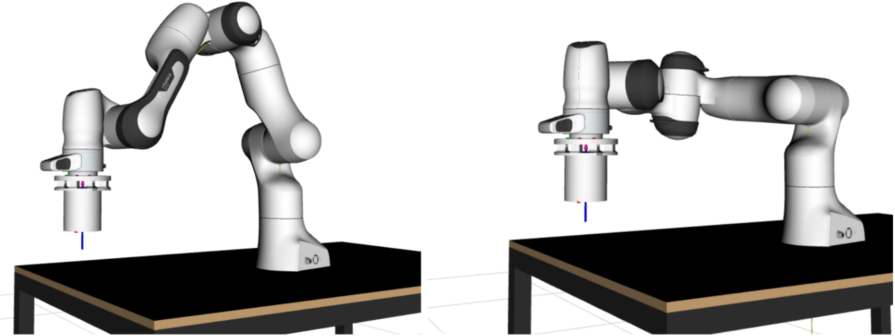

(a) Configuration $\boldsymbol{q}_1$ (b) Configuration $\boldsymbol{q}_2$

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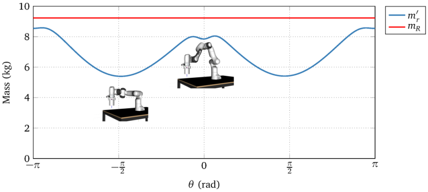

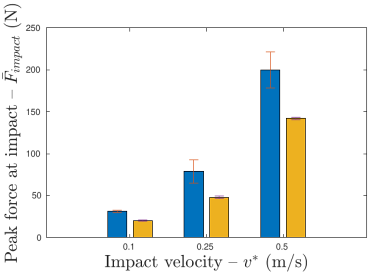

(b) Comparison of the averaged maximum peak force at impact time as a function of impact velocity and in two different configurations $\boldsymbol{q}_1$ (blue) and $\boldsymbol{q}_2$ (yellow). Standard deviation is plotted as a red whisker.

Outline of the presentation

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Important observations

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$$\int_u F_{impact}\,du = E_{dissipated} = E_c^{hum} + E_c^{rob}$$

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$$E_{c,k} = \frac{1}{2}\dot{\boldsymbol{x}}_k^T \boldsymbol{\Lambda}(\boldsymbol{q}_k)\dot{\boldsymbol{x}}_k \qquad \text{with} \qquad \boldsymbol{\Lambda}(\boldsymbol{q}) = (\boldsymbol{J}(\boldsymbol{q})\boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{J}^T(\boldsymbol{q}))^{-1}$$

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- Future Kinetic Energy : $E_{c,k+1} = E_{c,k} + \Delta E_c$

$$\Delta E_c = \underbrace{\left(\dot{\boldsymbol{x}}_k \Delta t + 0.5\ddot{\boldsymbol{x}}_{k+1}^c (\Delta t)^2\right)^T}_{\text{Expected task motion}} \underbrace{\boldsymbol{\Lambda}(\boldsymbol{q}_k)\ddot{\boldsymbol{x}}_{k+1}}_{\text{Equivalent actuation force}}$$

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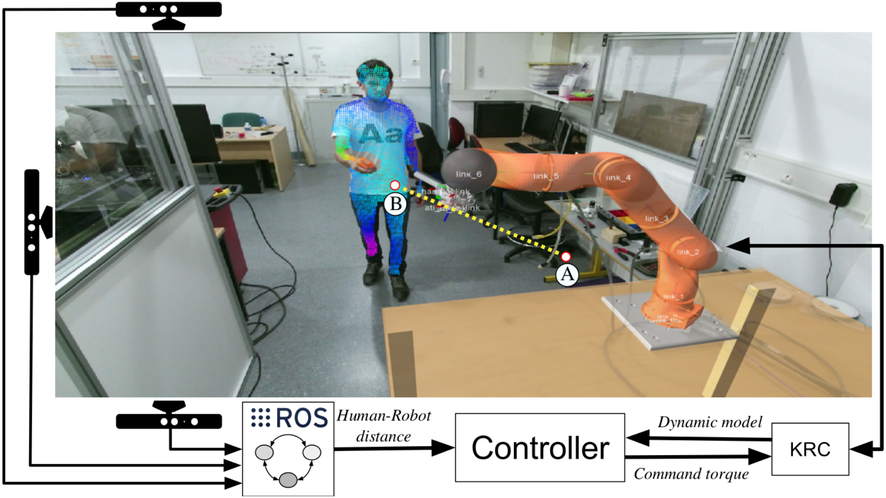
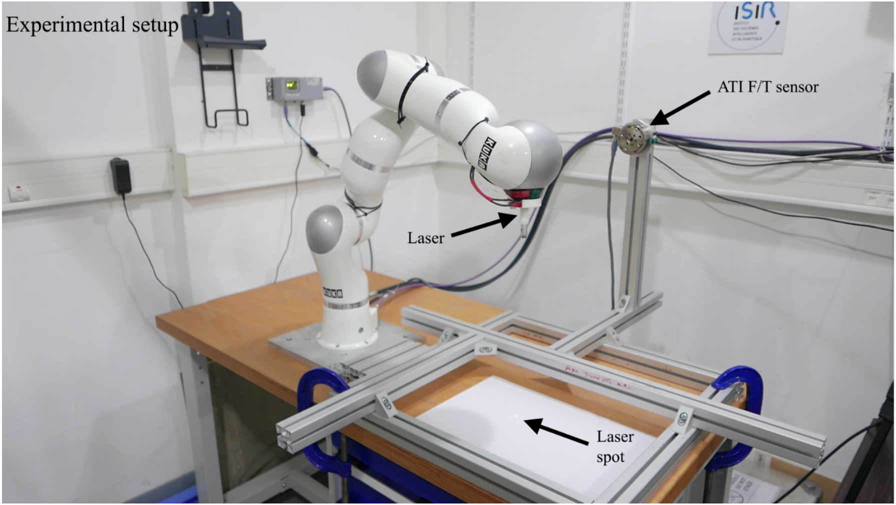
- We can write a constraint on Kinetic energy at each time [ISO 2016]

Energetic approach to safety



. [Meguenani 2017a],[Joseph 2018b]

Energetic approach to safety



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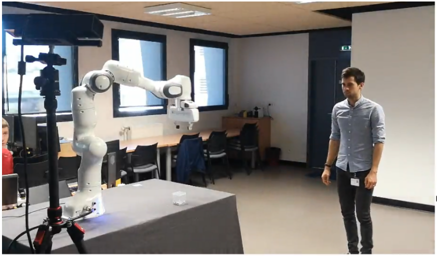
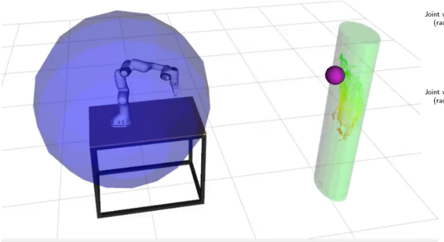
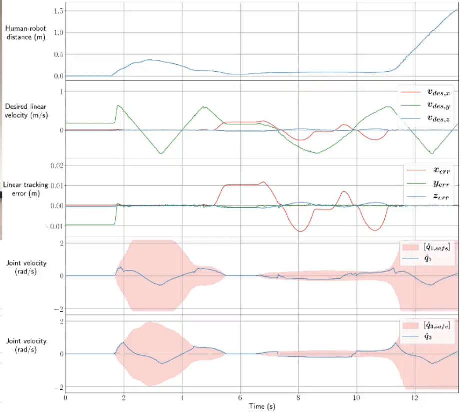
Energetic approach to safety [Joseph 2020]





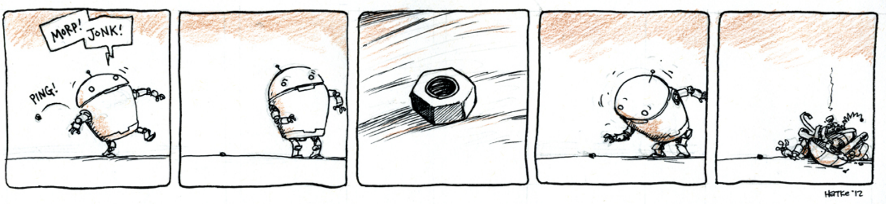


The robot safely continues operating when in proximity to the human.

Linear tracking error is minimized by modulating the desired linear velocity of the trajectory.

This allows to maximize the robot's performance without jeopardizing the human's safety

– To be continued –



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