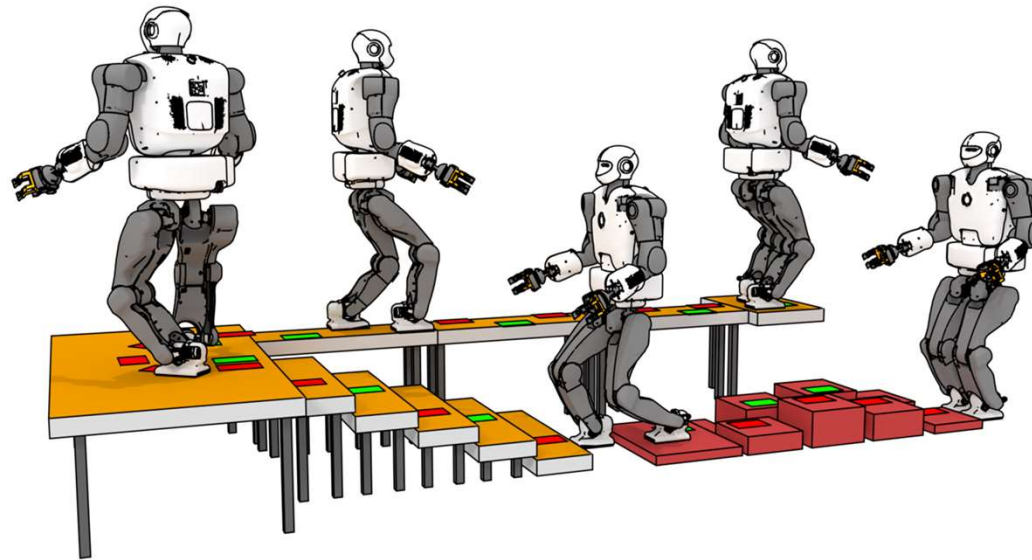


# The "contact planning problem" for legged robots: A cardinality minimisation approach

---



# Collaborators



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EWhA WOMANS UNIVERSITY



Andrea Del Prete



UNIVERSITY  
OF TRENTO - Italy



Nicolas Mansard



Pierre Fernbach



Solving Footstep Planning as a Feasibility Problem using L1-norm Minimization, *RAL 2021*

# Research goal: autonomous legged locomotion

---

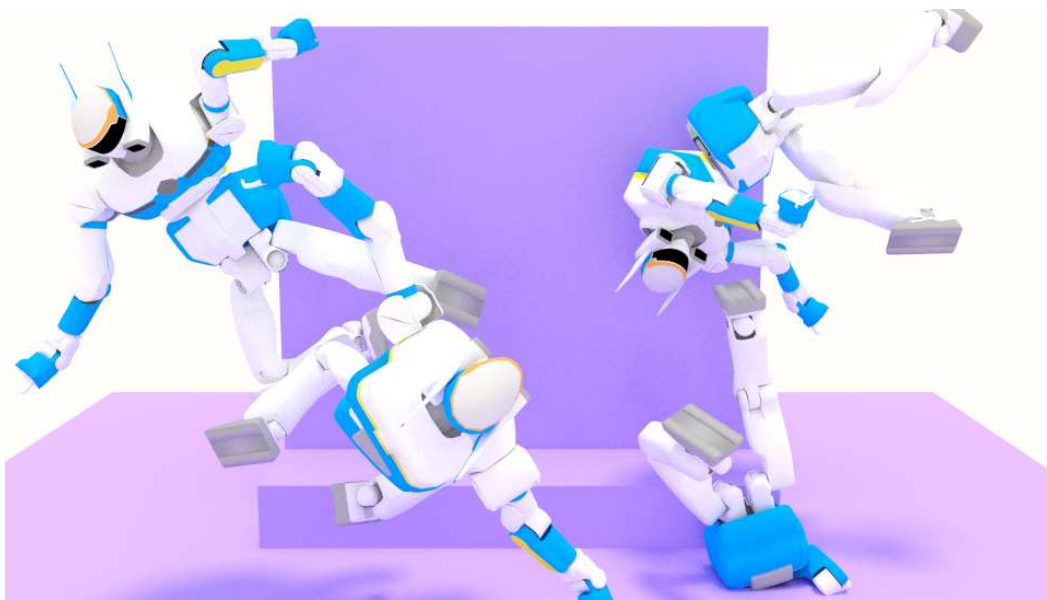


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Carpentier, Tonneau, Naveau, Stasse et Mansard, ICRA 16

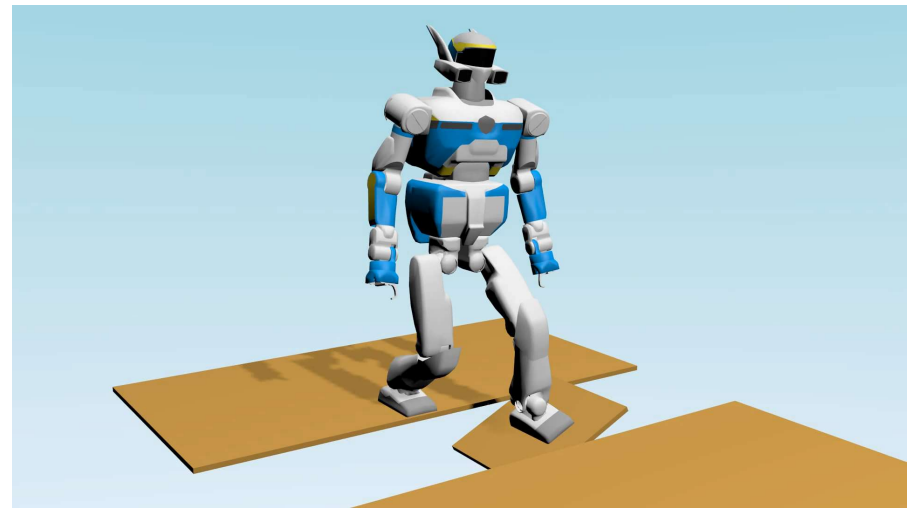
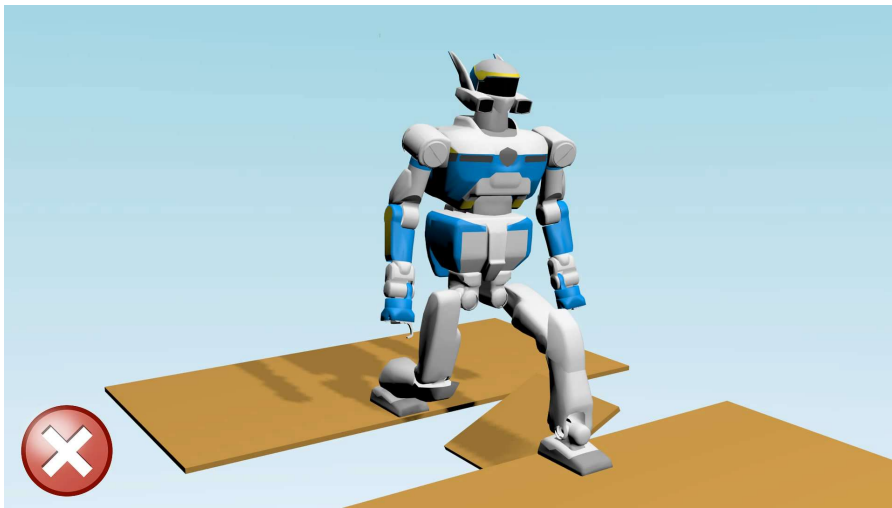
# Contact postures in high-dimensional space ?

---



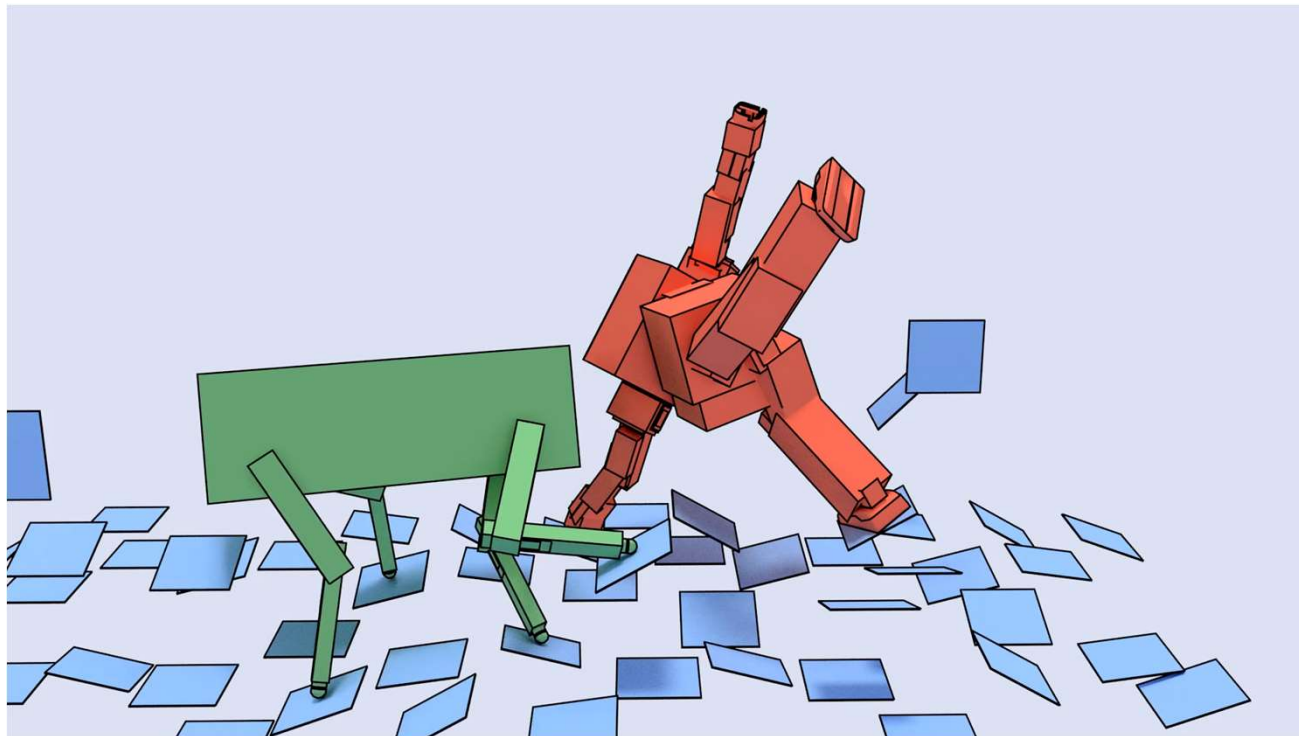
# Contact-dependent, discontinuous, non-linear dynamics / geometric constraints

---



# Contact interactions without collisions ?

---



# Legged locomotion is too hard

---

We need to cheat !

This has a cost ... not discussed today

---

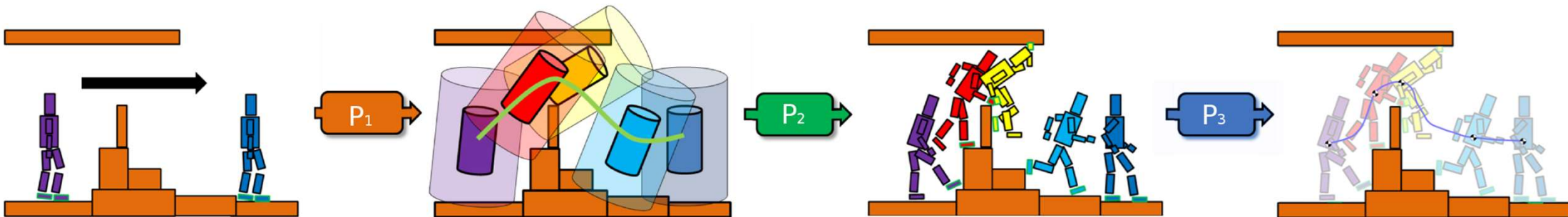
# A divide and conquer approach [Tonneau et al. 15]

---

Global path planner

Contact  
Planner

Whole body motion  
generator





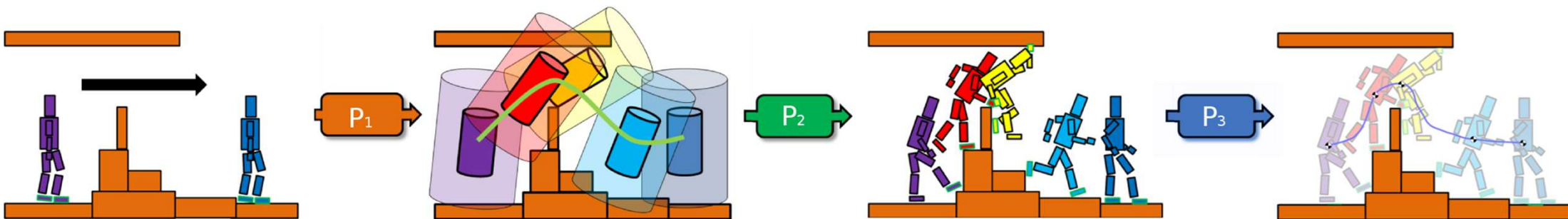
# A divide and conquer approach [Tonneau et al. 15]

---

Global path planner

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Planner

Whole body motion  
generator



---

Solving  $\mathcal{P}_i$  in the feasibility domain of  $\mathcal{P}_j, j > i$ ?

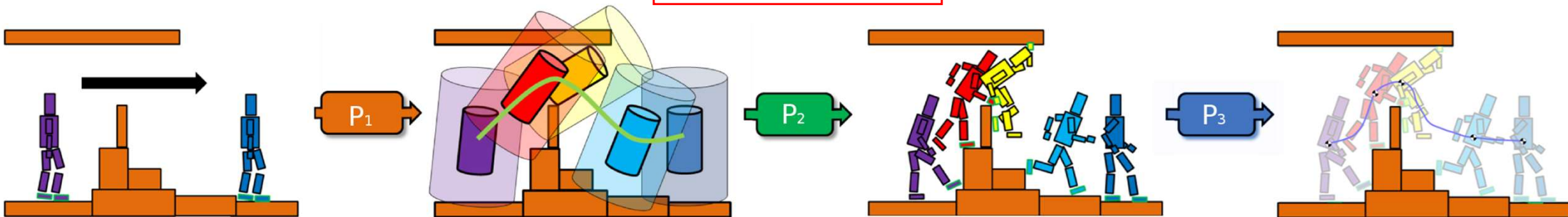
# A divide and conquer approach [Tonneau et al. 15]

---

Global path planner

Contact  
Planner

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generator

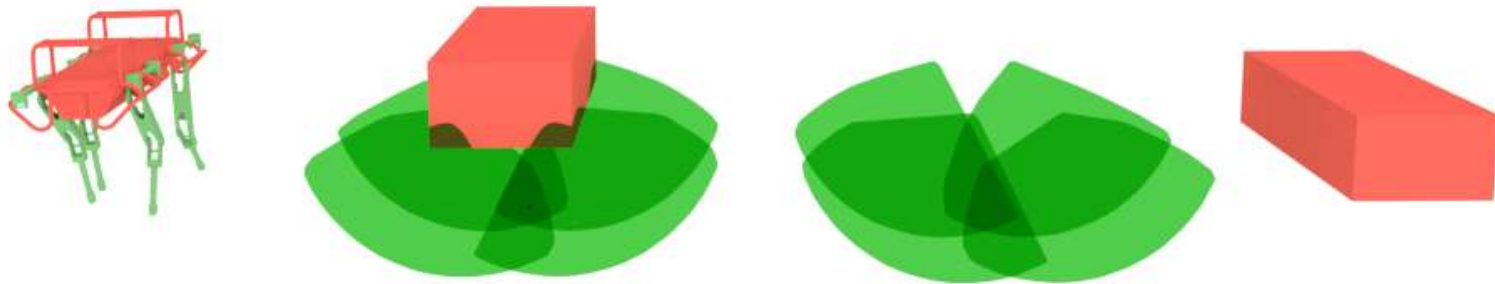


---

Solving  $\mathcal{P}_i$  in the feasibility domain of  $\mathcal{P}_j, j > i$ ?

# Cheating assumption: low-dimensional space

---

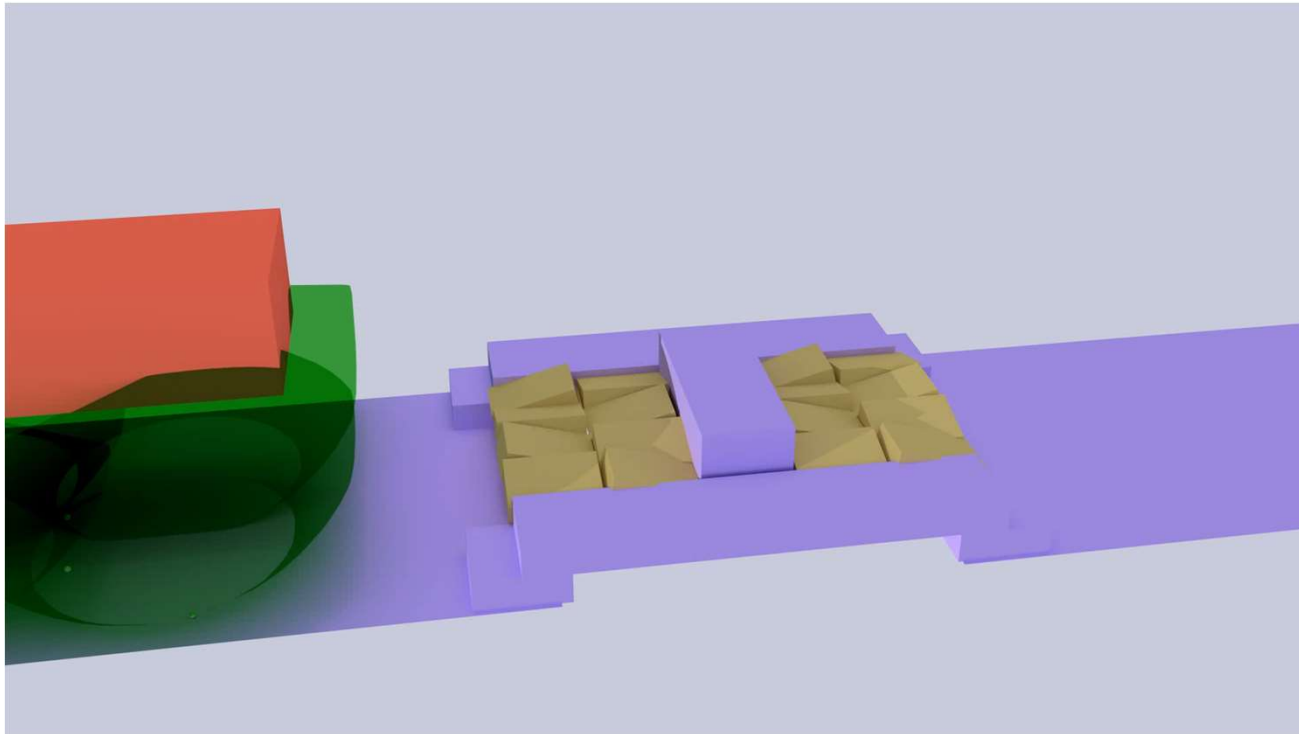


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Dimensionality reduction for contacts / collision constraints [Touneau et al. 2014]

# Cheating assumption: low-dimensional space

---

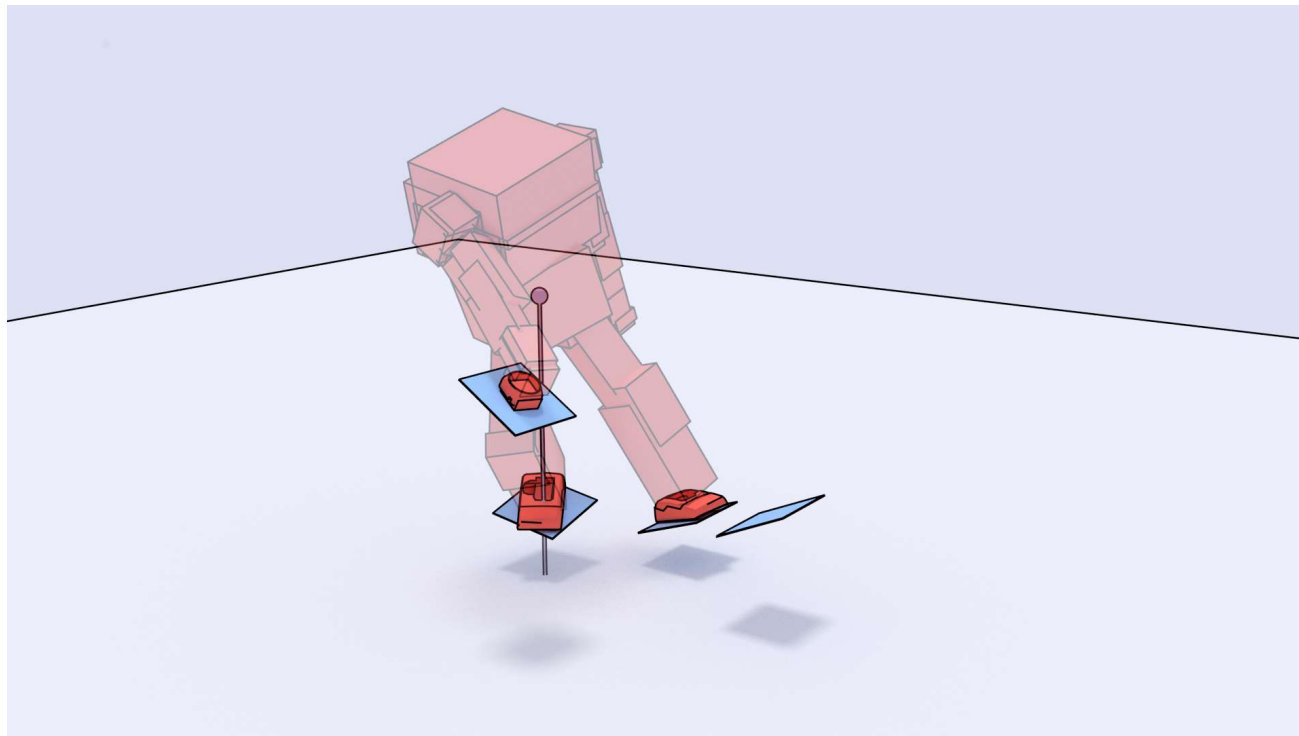


---

Low dimensional root path planner (RBPRM) [Tonneau et al. 15]

# Cheating assumption: low-dimensional space

---

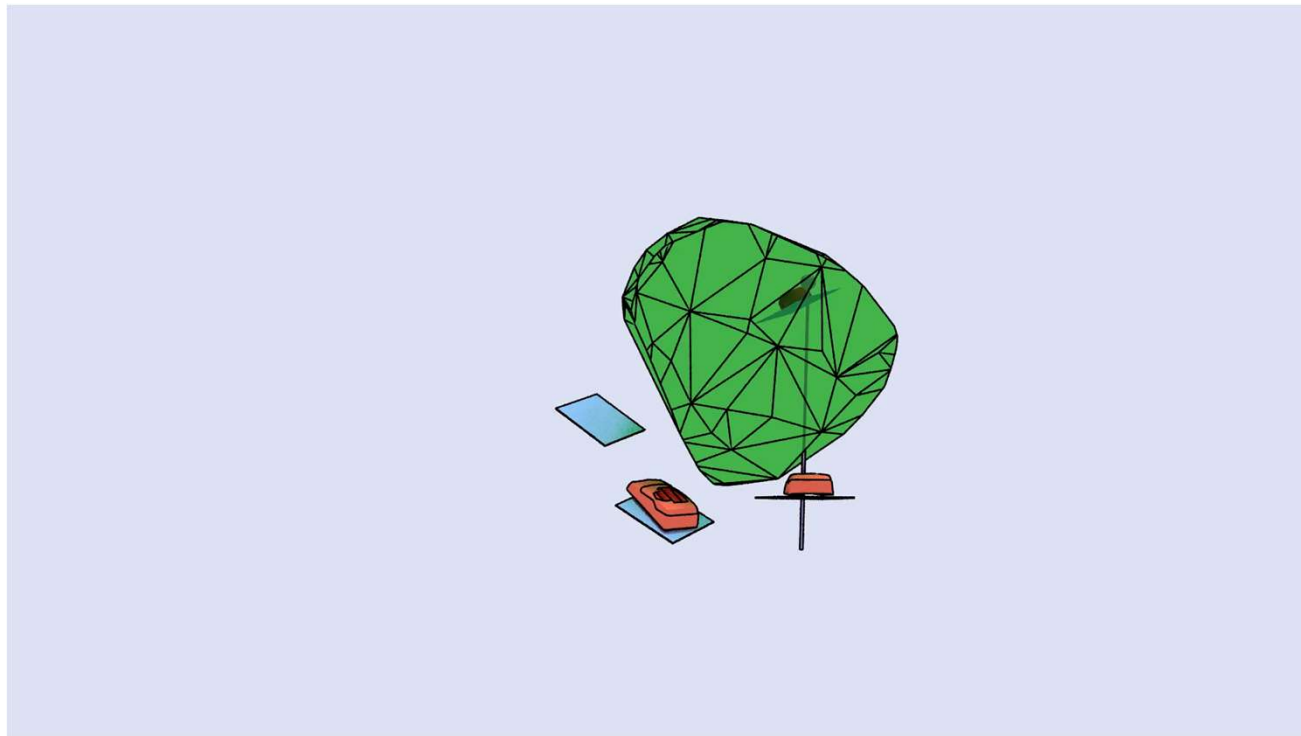


---

Dimensionality reduction (Centroidal model) [kajita 03, Orin 09]

# Cheating assumption: linear kinematic constraints

---



---

Linear geometric constraints [Tonneau et al. 18]

# ~~Cheat..~~ conservative assumption: linear dynamics

---



---

CROC [Fernbach, Tonneau et al. 2020]

# What's left ?

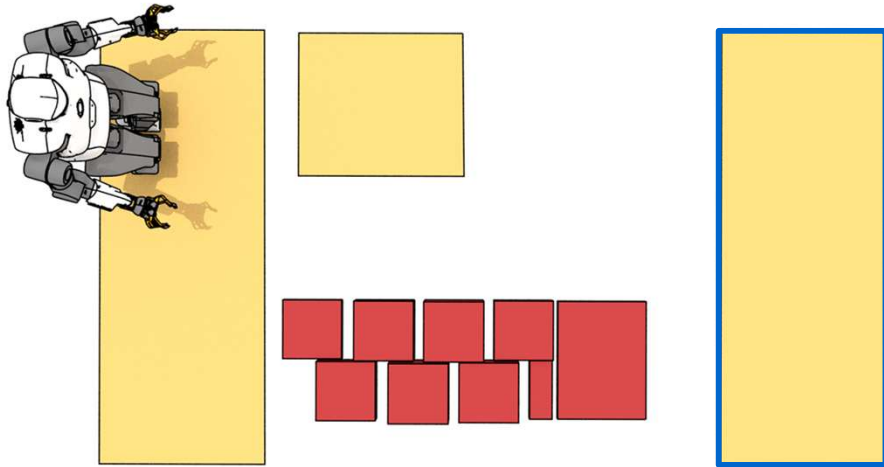
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A combinatorial problem of large polynomial complexity...



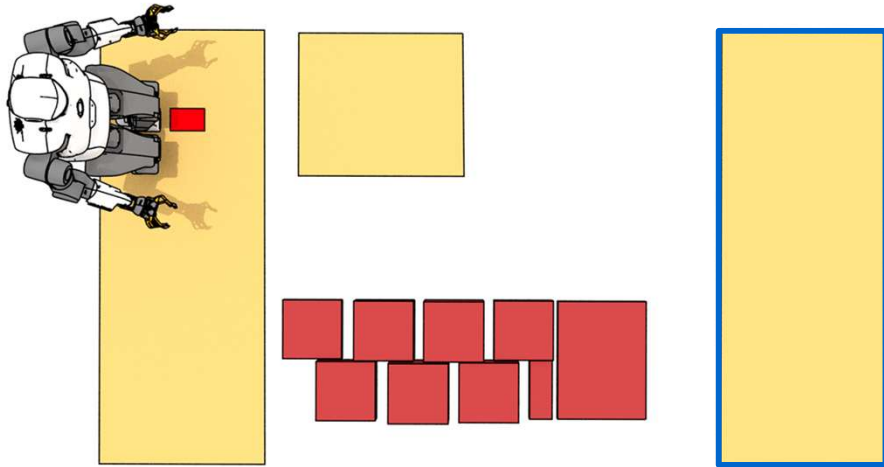
# Example: how to reach the platform ?

---



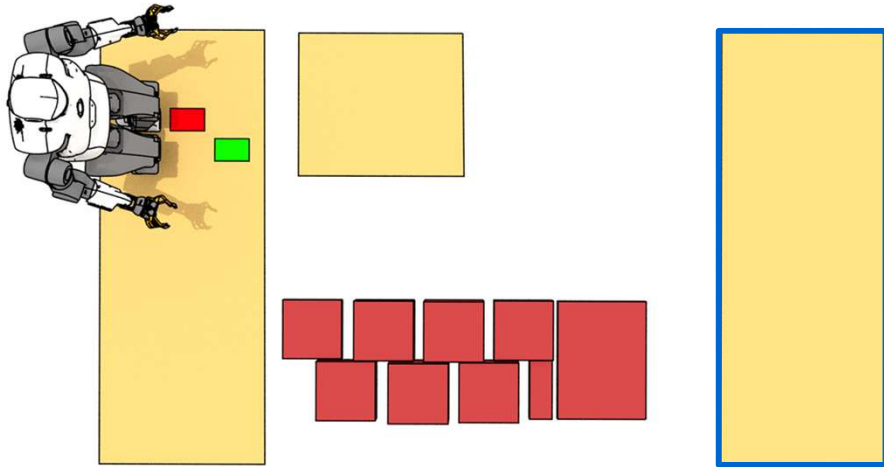
# Combinatorial step planning

---



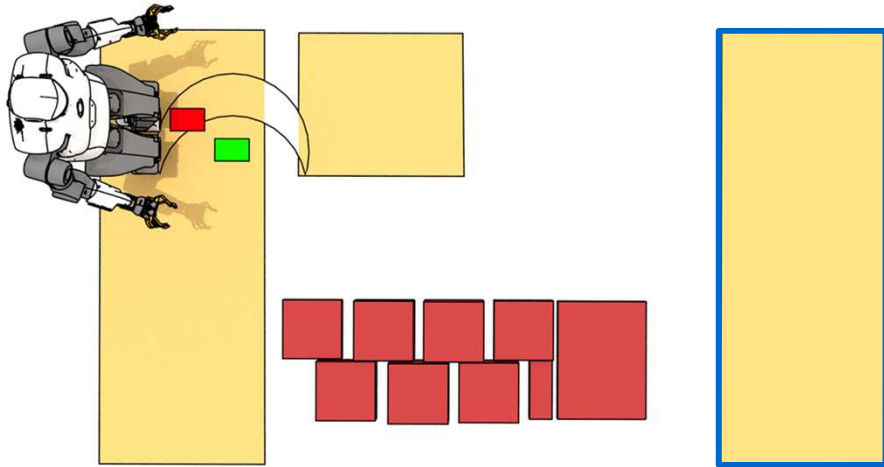
# Combinatorial step planning

---



# Combinatorial step planning

---

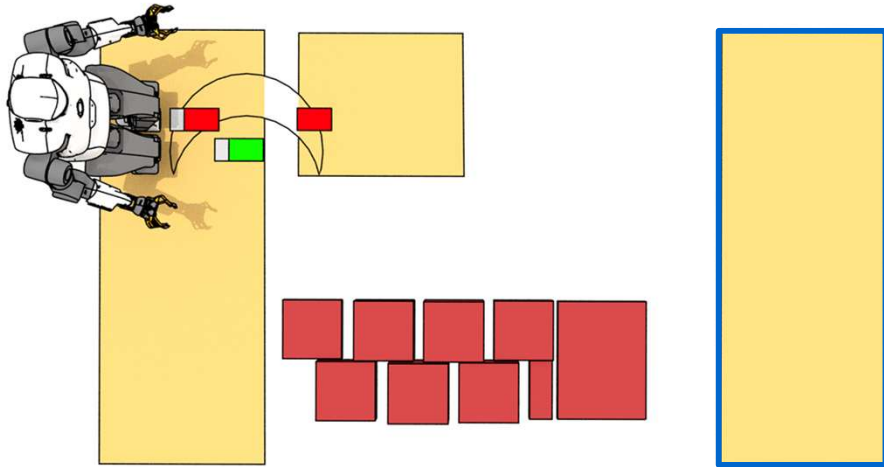


Feasibility  $\mathcal{F}$  :

Geometric constraints

# Combinatorial step planning

---

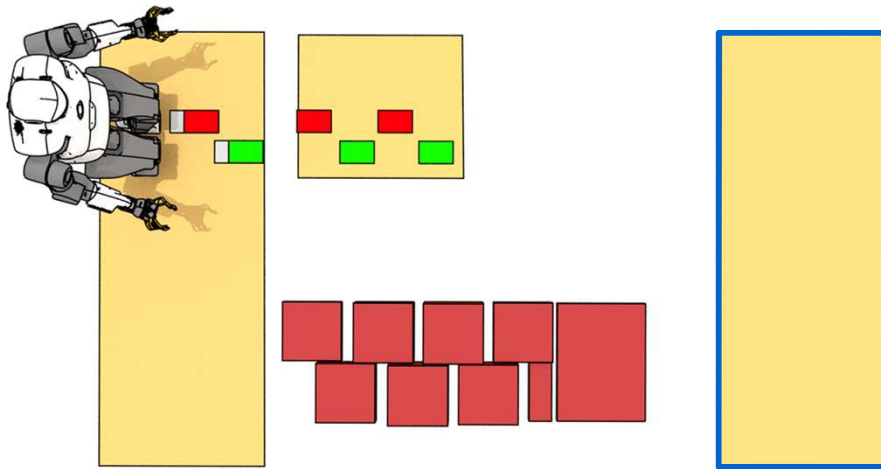


Feasibility  $\mathcal{F}$  :

Geometric constraints

# Combinatorial step planning

---



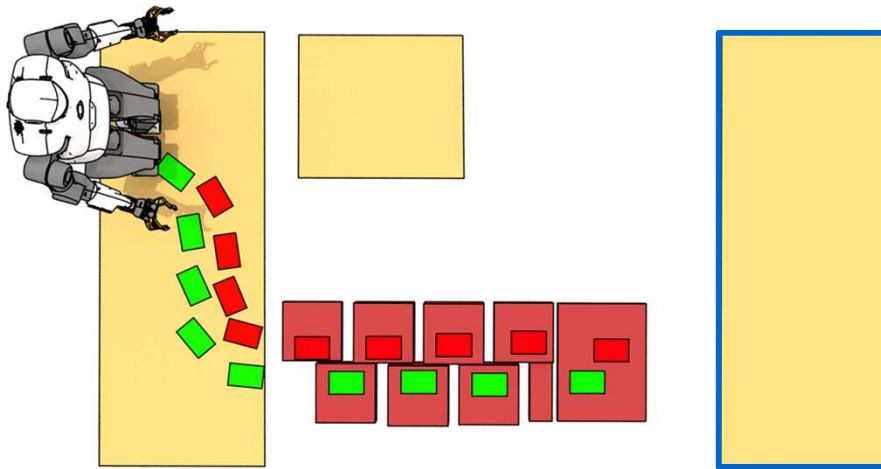
Global path search

Feasibility  $\mathcal{F}$  :

Geometric constraints

# Combinatorial step planning

---



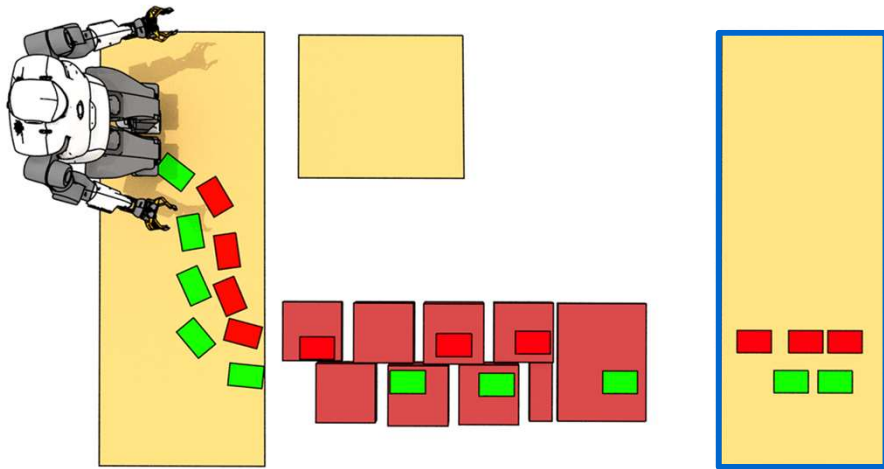
Global path search

Feasibility  $\mathcal{F}$  :

Geometric constraints

# Combinatorial step planning

---



Global path search

Feasibility  $\mathcal{F}$  :

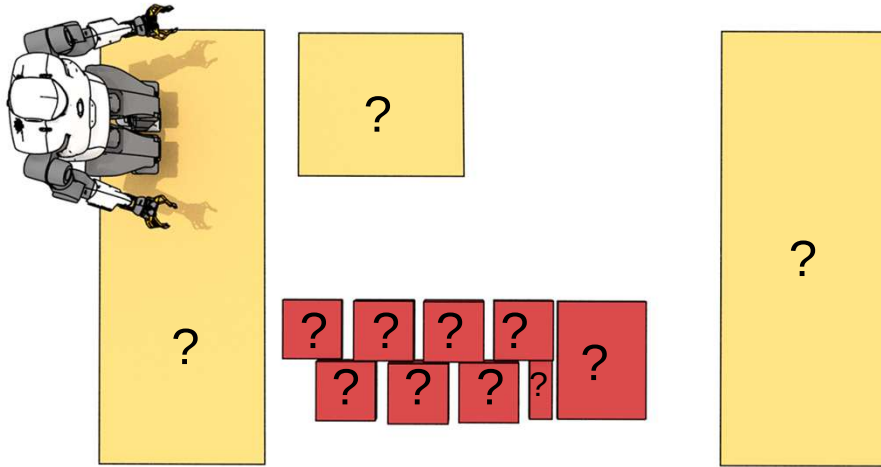
Geometric constraints

Dynamic constraints



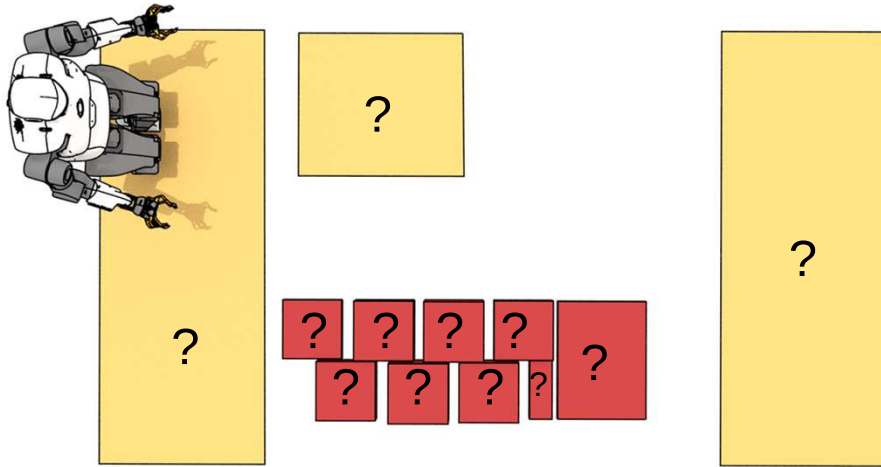
# Which contact surface ?

---



# Which contact surface ?

---



Convex contact surfaces:

$$\mathcal{S}_j : \{ \mathbf{p}, \mathbf{S}_j \mathbf{p} \leq \mathbf{s}_j \}$$

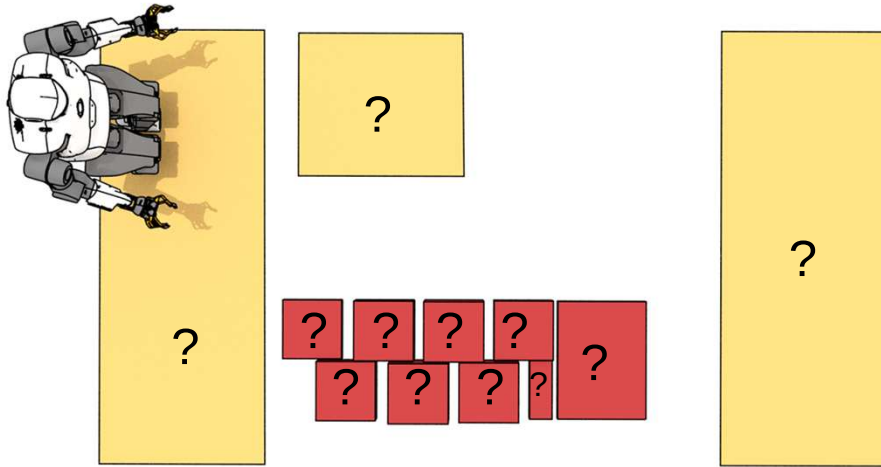
$$1 \leq j \leq n$$

Footsteps positions:

$$\mathbf{p}_i, 1 \leq i \leq m$$

# Which contact surface ?

---



Convex contact surfaces:

$$\mathcal{S}_j : \{ \mathbf{p}, \mathbf{S}_j \mathbf{p} \leq \mathbf{s}_j \}$$

$$1 \leq j \leq n$$

Footsteps positions:

$$\mathbf{p}_i, 1 \leq i \leq m$$

Combinatorial:

$$m^n$$

# Contact planning as a feasibility problem

---

find  $\mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$

s.t.  $\mathbf{X} \in \mathcal{F}$  // Feasibility

$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$  // Initial and goal conditions

$\forall i :$

$\mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n$

# Contact planning as a feasibility problem

---

find  $\mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$

s.t.  $\mathbf{X} \in \mathcal{F}$  // Feasibility

$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$  // Initial and goal conditions

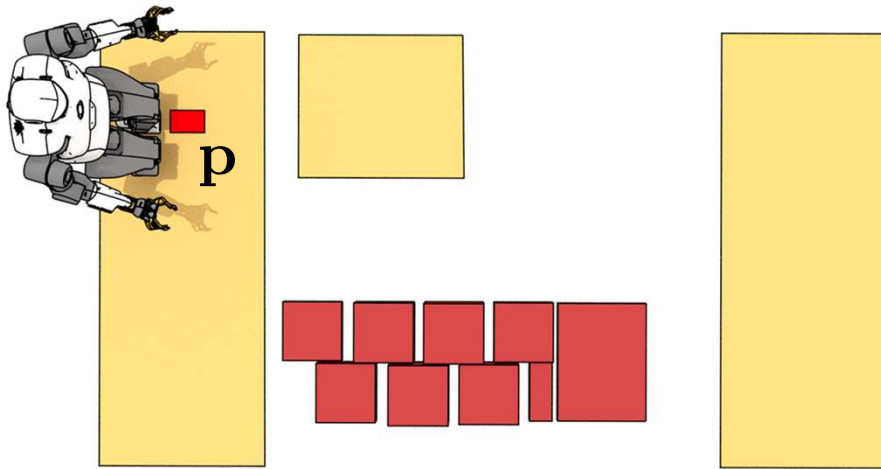
$\forall i :$

$$\mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n$$

How to tackle the combinatorics ?

# An equivalent cardinality minimisation problem

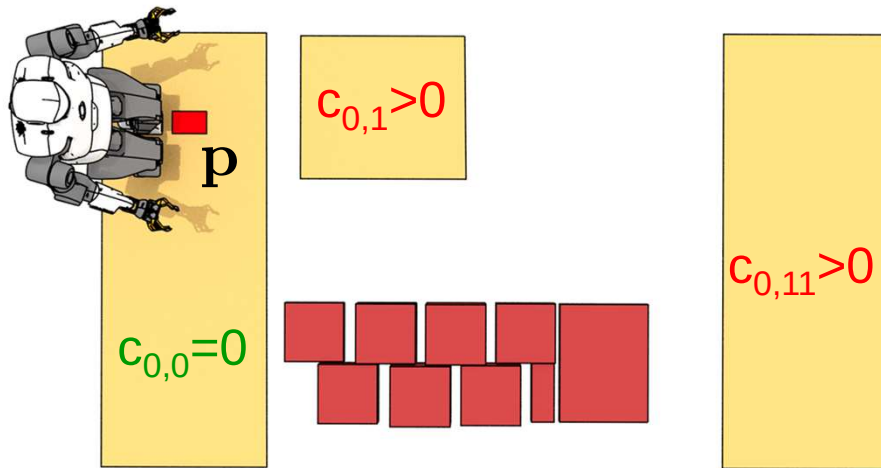
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$$S_j p_i \leq s_j \Leftrightarrow p_i \in \mathcal{S}_j$$

# An equivalent cardinality minimisation problem

---

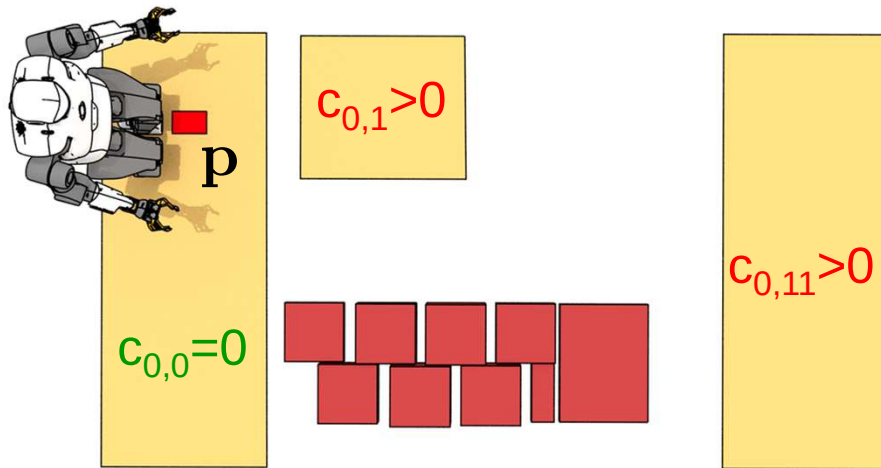


Slack variables  $c_{i,j} \in \mathbb{R}^+$

$$\left. \begin{array}{l} \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \\ c_{i,j} = 0 \end{array} \right\} \Rightarrow \mathbf{p}_i \in \mathcal{S}_j$$
$$\mathbf{c}_i = [c_{i,1}, \dots, c_{i,n}]$$

# An equivalent cardinality minimisation problem

---



Slack variables  $c_{i,j} \in \mathbb{R}^+$

$$\left. \begin{array}{l} \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \\ c_{i,j} = 0 \end{array} \right\} \Rightarrow \mathbf{p}_i \in \mathcal{S}_j$$

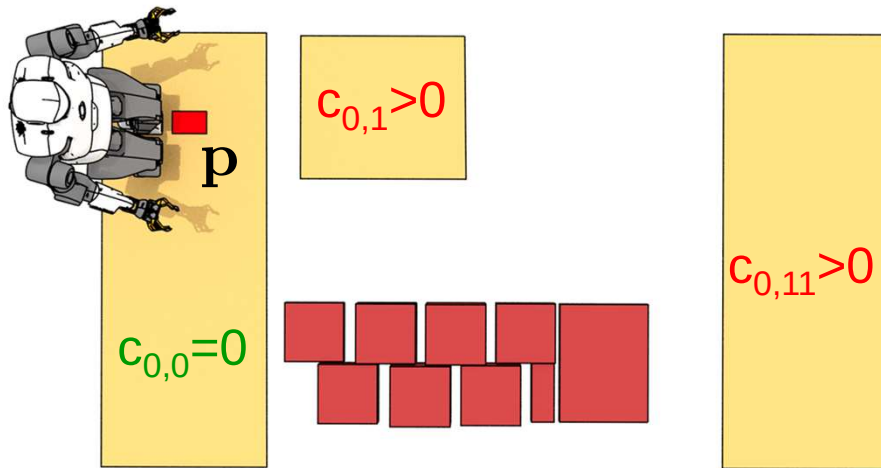
$$\mathbf{c}_i = [c_{i,1}, \dots, c_{i,n}]$$

$$\#NonZeros(\mathbf{c}_i) = n - 1 \Rightarrow \mathbf{p}_i \text{ on a surface}$$

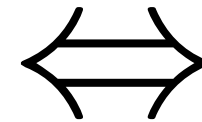


# An equivalent cardinality minimisation problem

---



find  $\mathbf{p}_i$   
s.t.  $\mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n$



find  $\mathbf{p}_i, \mathbf{c}_i$   
min  $card(\mathbf{c}_i)$   
s.t.  $\mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \quad \forall j$

# An equivalent cardinality minimisation problem

---

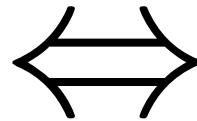
find  $\mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$

s.t.  $\mathbf{X} \in \mathcal{F}$

$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$

$\forall i :$

$$\mathbf{p}_i \in \mathcal{S}_1 \vee \dots \vee \mathbf{p}_i \in \mathcal{S}_n$$



find  $\mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$

$\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n]$

min  $\sum_{i=1}^m \text{card}(\mathbf{c}_i)$

s.t.  $\mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j$

$\mathbf{X} \in \mathcal{F}$

$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$

# Convex relaxation with sparsity inducing norm

---

$$\text{find } \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$$

$$\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n]$$

$$\text{min } \sum_{i=1}^m \text{card}(\mathbf{c}_i)$$

$$\text{s.t. } \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j$$

$$\mathbf{X} \in \mathcal{F}$$

$$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$$

$\approx$

$$\text{find } \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$$

$$\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n]$$

$$\text{min } \sum_{i=1}^m \|\mathbf{c}_i\|_1$$

$$\text{s.t. } \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j$$

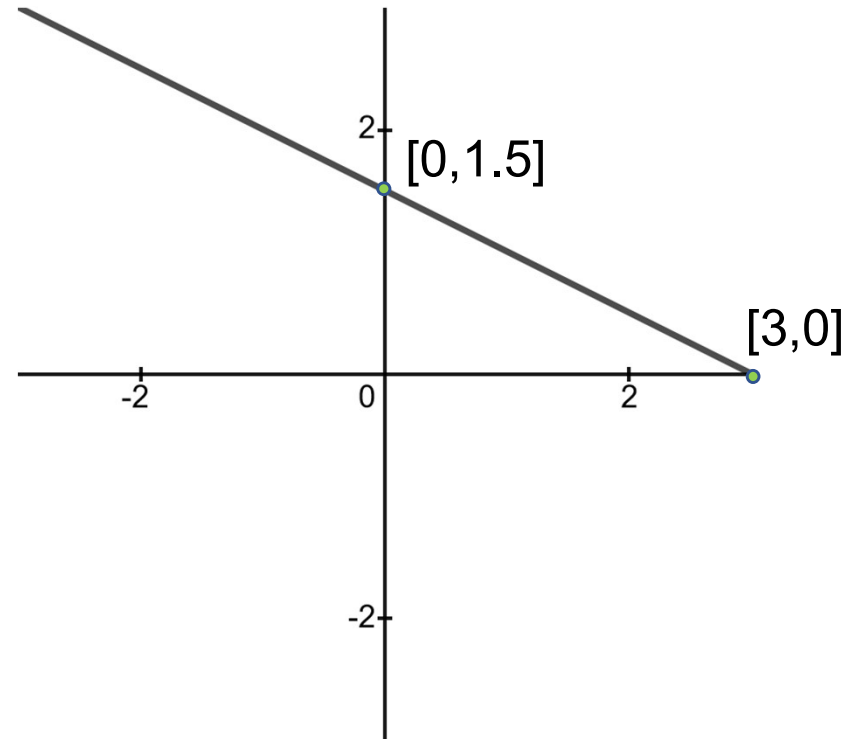
$$\mathbf{X} \in \mathcal{F}$$

$$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$$

# L1 Norm induces sparsity

---

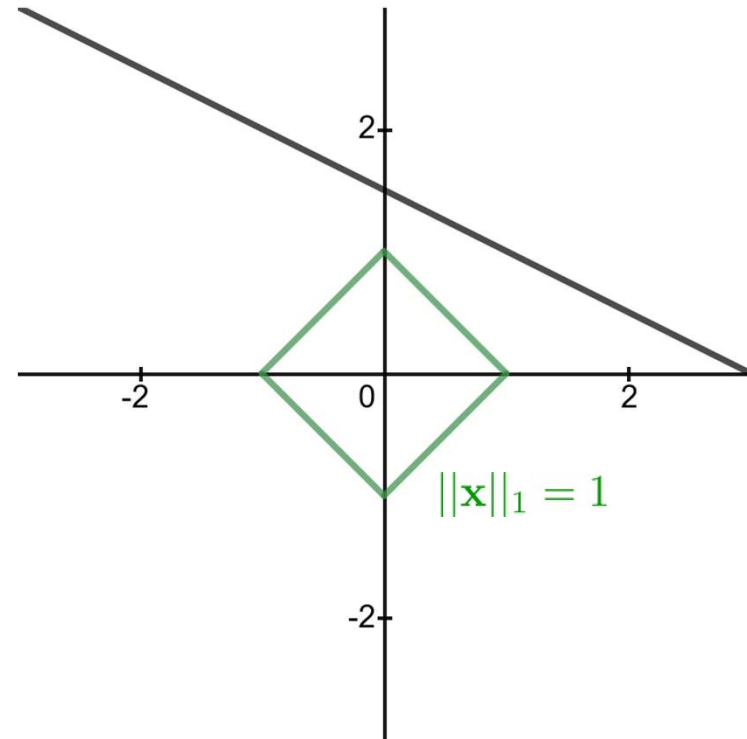
find  $\mathbf{x} = [x_1, x_2]$   
min  $card(\mathbf{x})$   
s.t.  $x_2 = 1.5 - 0.5 x_1$



# L1 Norm induces sparsity

---

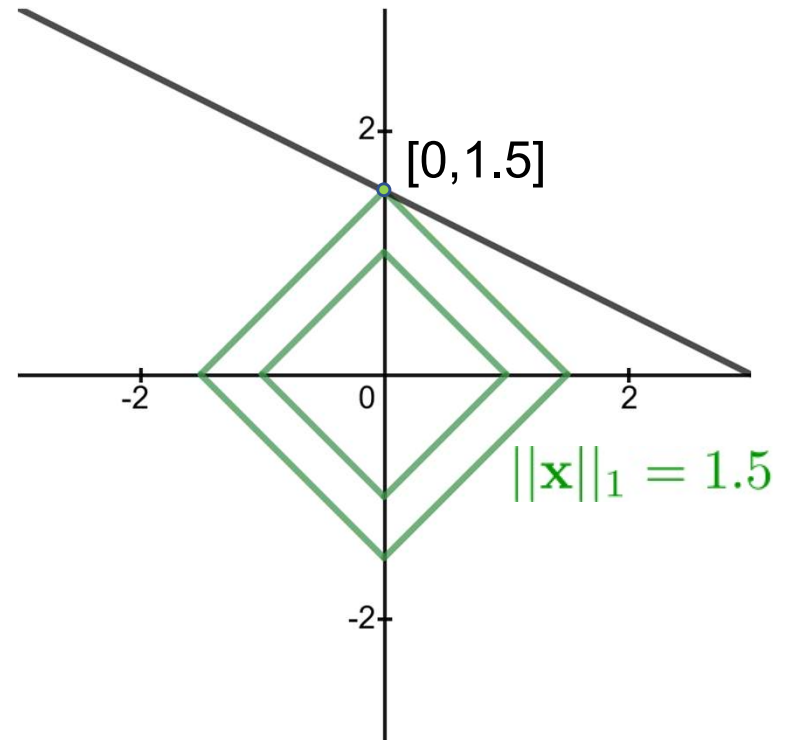
$$\begin{array}{ll} \text{find} & \mathbf{x} = [x_1, x_2] \\ \text{min} & \cancel{\text{card}(\mathbf{x})} \|\mathbf{x}\|_1 \\ \text{s.t.} & x_2 = 1.5 - 0.5 x_1 \end{array}$$



# L1 Norm induces sparsity

---

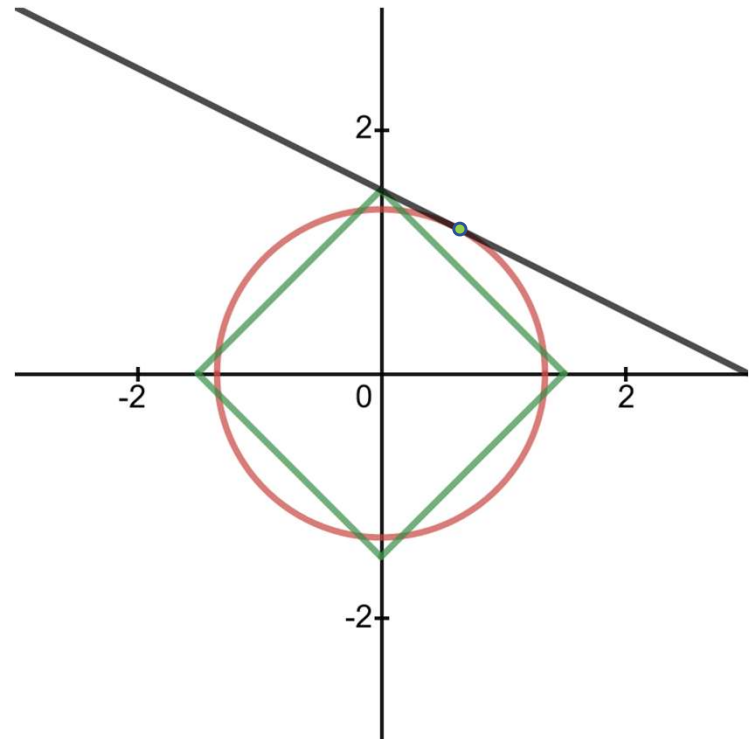
$$\begin{array}{ll} \text{find} & \mathbf{x} = [x_1, x_2] \\ \text{min} & \cancel{\text{card}(\mathbf{x})} \|\mathbf{x}\|_1 \\ \text{s.t.} & x_2 = 1.5 - 0.5 x_1 \end{array}$$



# Euclidian norm does not lead to sparsity

---

$$\begin{array}{ll} \text{find} & \mathbf{x} = [x_1, x_2] \\ \text{min} & \cancel{\text{card}(\mathbf{x})} \|\mathbf{x}\|_2 \\ \text{s.t.} & x_2 = 1.5 - 0.5 x_1 \end{array}$$



# Convex relaxation with sparsity inducing norm

---

$$\text{find } \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$$

$$\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n]$$

$$\text{min } \sum_{i=1}^m \text{card}(\mathbf{c}_i)$$

$$\text{s.t. } \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j$$

$$\mathbf{X} \in \mathcal{F}$$

$$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$$

$$\approx$$

$$\text{find } \mathbf{X} = [\mathbf{p}_1 \dots \mathbf{p}_m]$$

$$\mathbf{C} = [\mathbf{c}_1 \dots \mathbf{c}_n]$$

$$\text{min } \sum_{i=1}^m \|\mathbf{c}_i\|_1$$

$$\text{s.t. } \mathbf{S}_j \mathbf{p}_i - \mathbf{1} c_{i,j} \leq \mathbf{s}_j \forall i, \forall j$$

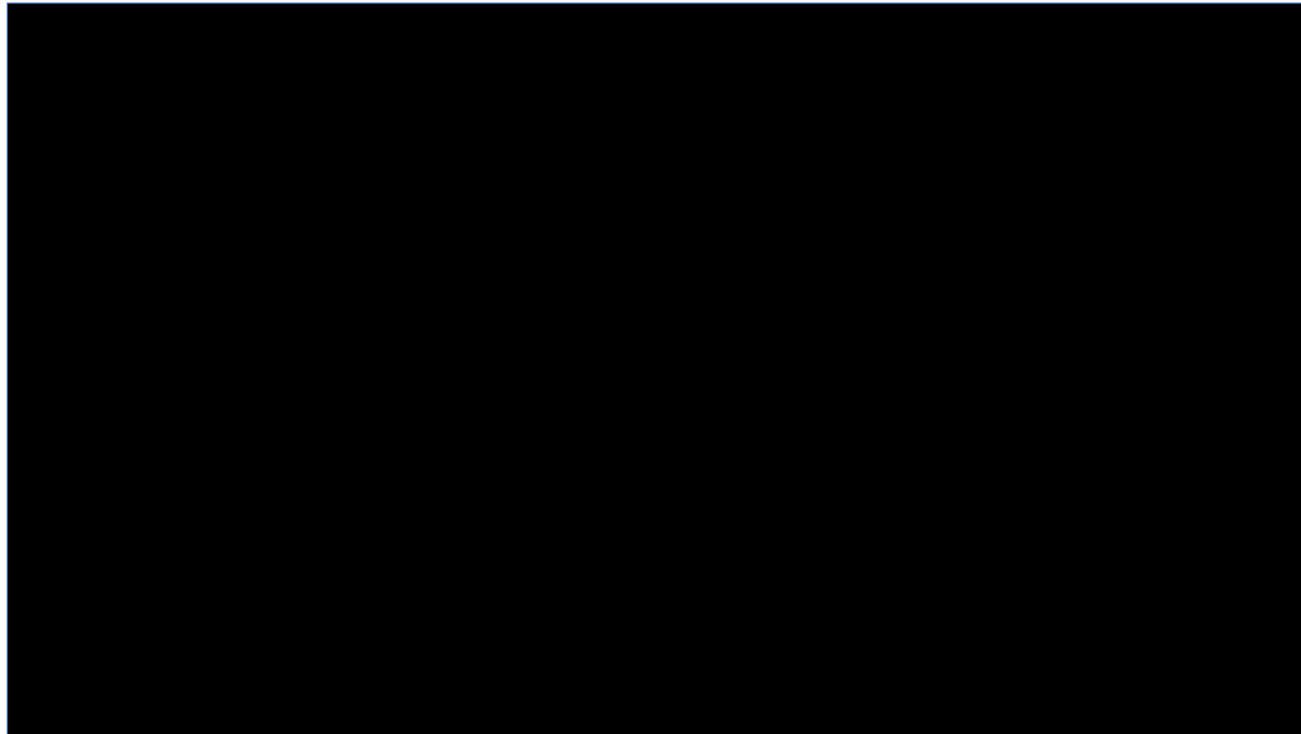
$$\mathbf{X} \in \mathcal{F}$$

$$\mathbf{X} \in \mathcal{I} \cap \mathcal{G}$$



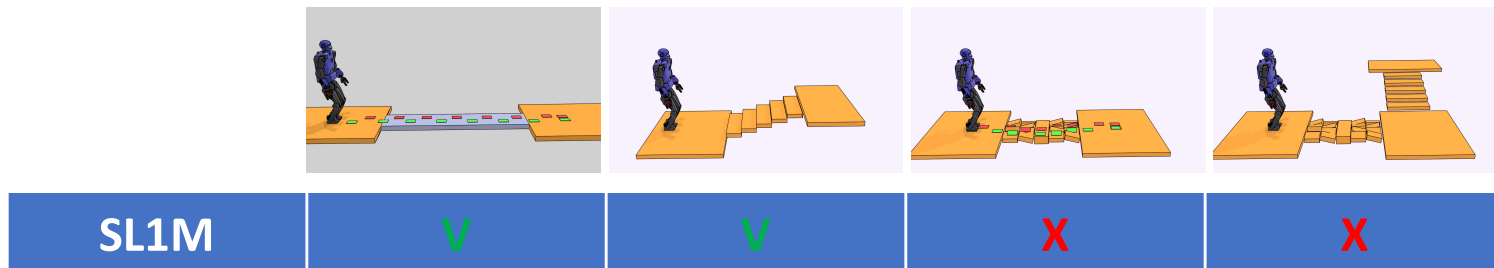
Does it really work ?

---



# Does it really work ?

---



# L1 relaxation is faster, but less reliable

---

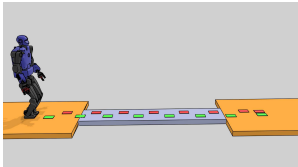
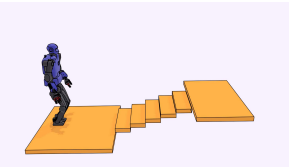
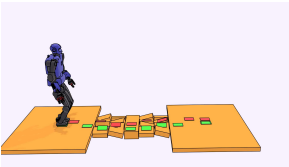
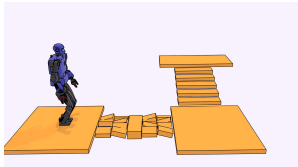
L1 up to 10x faster than Mixed-Integer (MI) solvers...

What if relaxed problem does not converge to sparse solution ?

- Can we guarantee convergence ?
- A more general question: Is this really a combinatorial problem?

# Let's look at the number of nodes explored by MI:

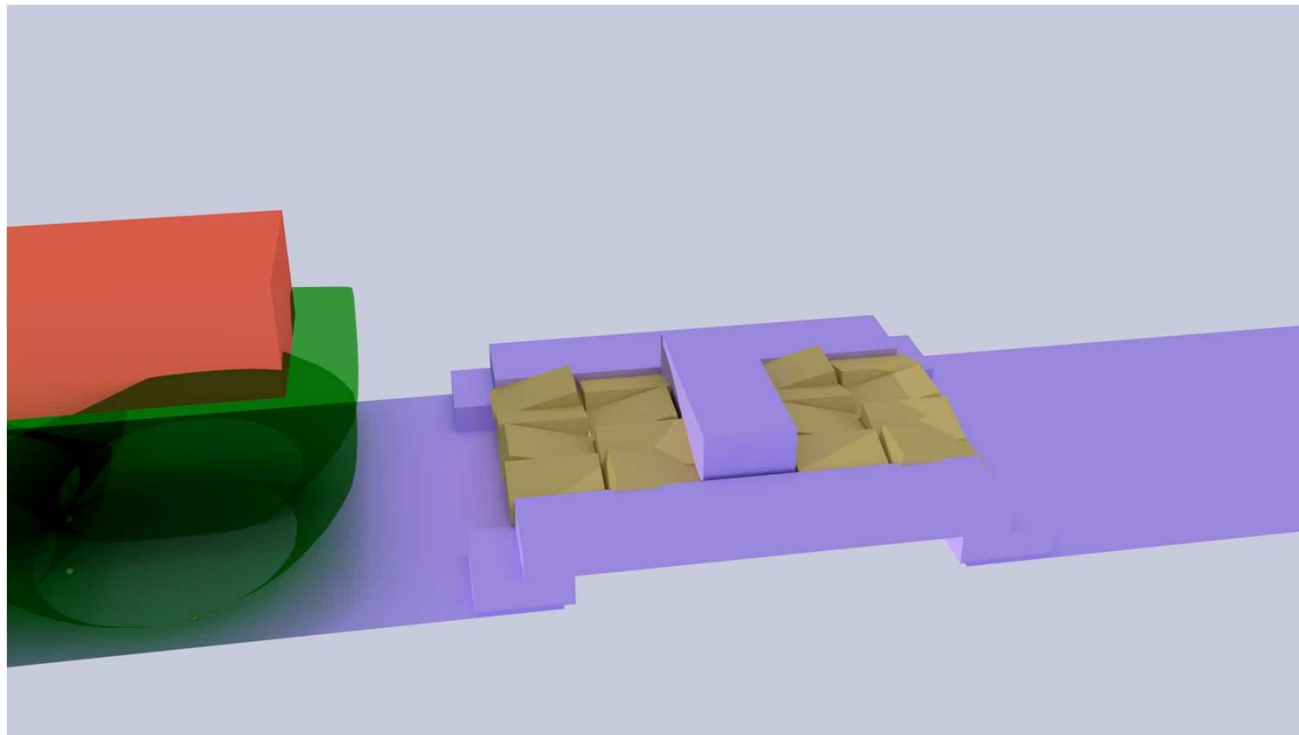
---

				
<b>MIP (+cost)</b>	<b>0 (1)</b>	<b>0 (1)</b>	<b>383 (2742)</b>	<b>2724 (20452)</b>
<b>SL1M</b>	<b>V</b>	<b>V</b>	<b>X</b>	<b>X</b>

Let's do some pruning !

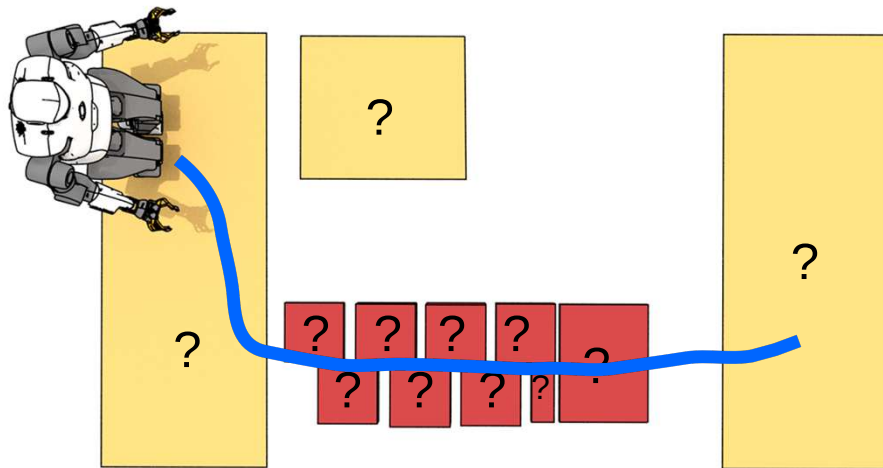
# COMPLEXITY CAN BE REDUCED WITH A GUIDE

---



# COMPLEXITY CAN BE REDUCED WITH A GUIDE

---



Contact surfaces

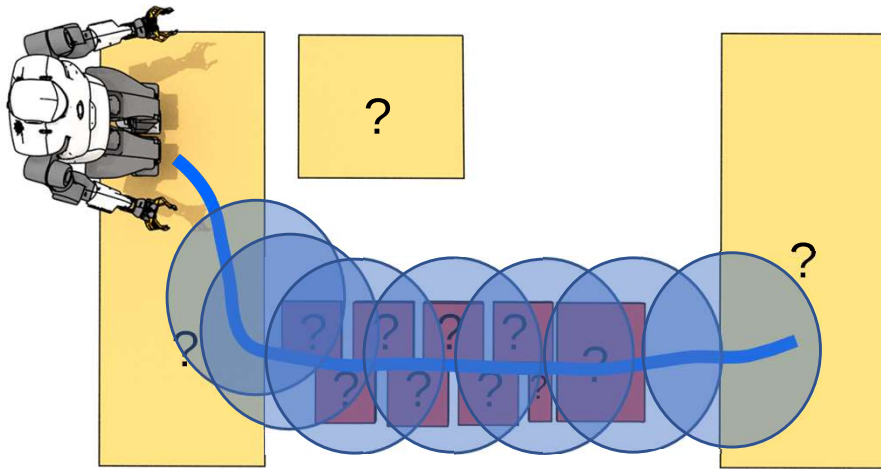
$$\mathcal{S}_j, 1 \leq j \leq n$$

Steps:

$$\mathbf{p}_i, 1 \leq i \leq m$$

# COMPLEXITY CAN BE REDUCED WITH A GUIDE

---



Contact surfaces

$$\mathcal{S}_j, 1 \leq j \leq n$$

Steps:

$$\mathbf{p}_i, 1 \leq i \leq m$$

Combinatorial:

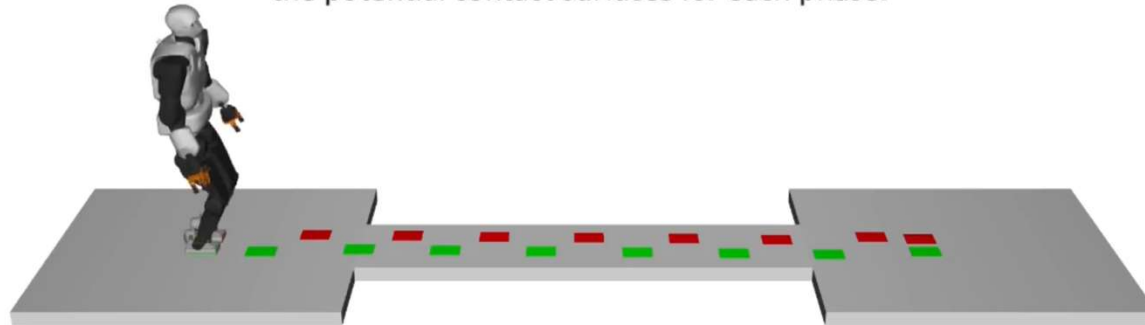
$$n^h, h < m$$

# Only hyper parameter is discretisation step

---



We integrate a sampling-based trajectory planner to automatically compute the potential contact surfaces for each phase.

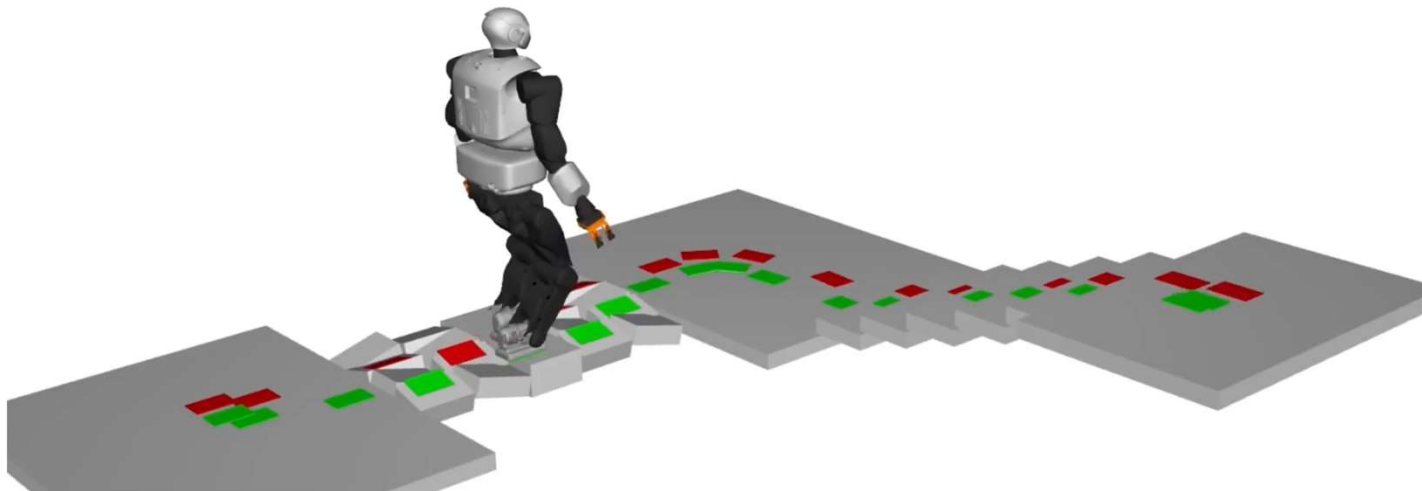




# Does it really work ?

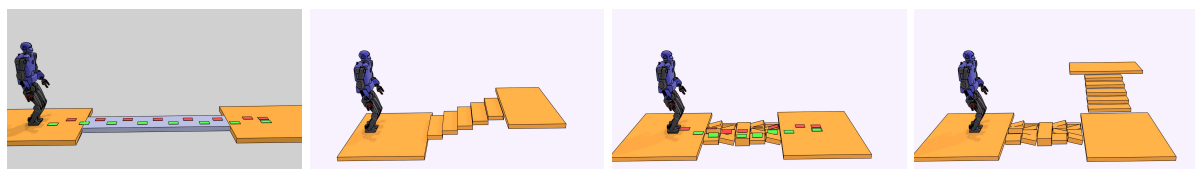
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Our footstep planning framework for legged robots is capable of computing 30-step long contact sequences on uneven terrain within a second.



# Guide-path improves SL1M convergence, but also MIP performances

---

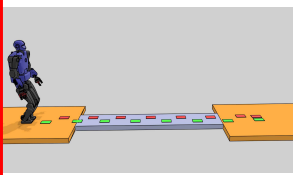
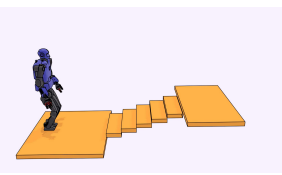
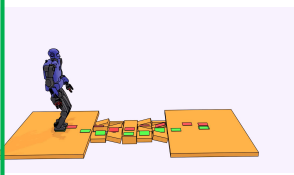
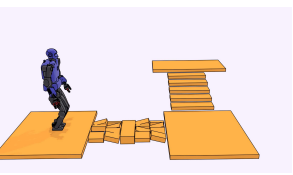


<b>L1 success</b>	V	V	X	X
<b>Guide + L1</b>	V	V	V	V

# Ok but why bother ? Computational time (ms)

---

Tradeoff guide cost (about 100 ms) / Solver time

				
<b>Guide + L1</b>	<b>35</b>	<b>20</b>	<b>73</b>	<b>219</b>
<b>Guide + MIP</b>	<b>77</b>	<b>34</b>	<b>230</b>	<b>508</b>
<b>MIP</b>	<b>166</b>	<b>59</b>	<b>389</b>	<b>74710</b>

# Early conclusions for footstep planning

---

- Refining problem definition leads to vanishing combinatorics
  - Using domain-specific knowledge seems promising
  - Other issues to consider (what is the right path discretisation ?)
- L1-norm relaxation converges to feasible solution faster
- What about more challenging problems (gait selection) ?
- Optimality ? I think we do not care!

# Overall conclusions for Contact planning

---

Should we embrace combinatorics or try to ignore it ?

What can we expect for more challenging tasks ?

ie. Can we « presolve » harder problems ?

Non-linear problems, unknown gait patterns

I believe those questions are challenging, important and fun

# That's all for now

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<https://stevetonneau.fr> for papers, videos, source code

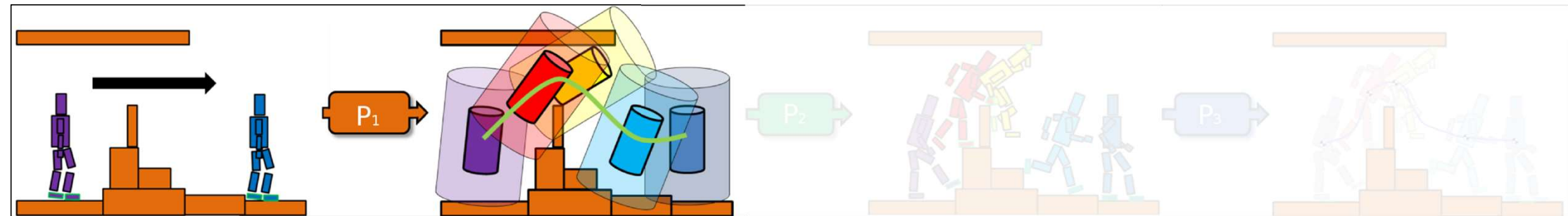
## II. Learning feasible guide trajectories

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Global path planner

Contact  
Planner

Whole body motion  
generator



# Collaborators

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Jason Chemin



Nicolas Mansard

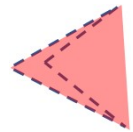


Learning to steer a locomotion contact planner, *ICRA 2021*

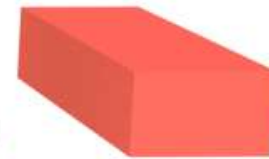


# Necessary condition for contact creation

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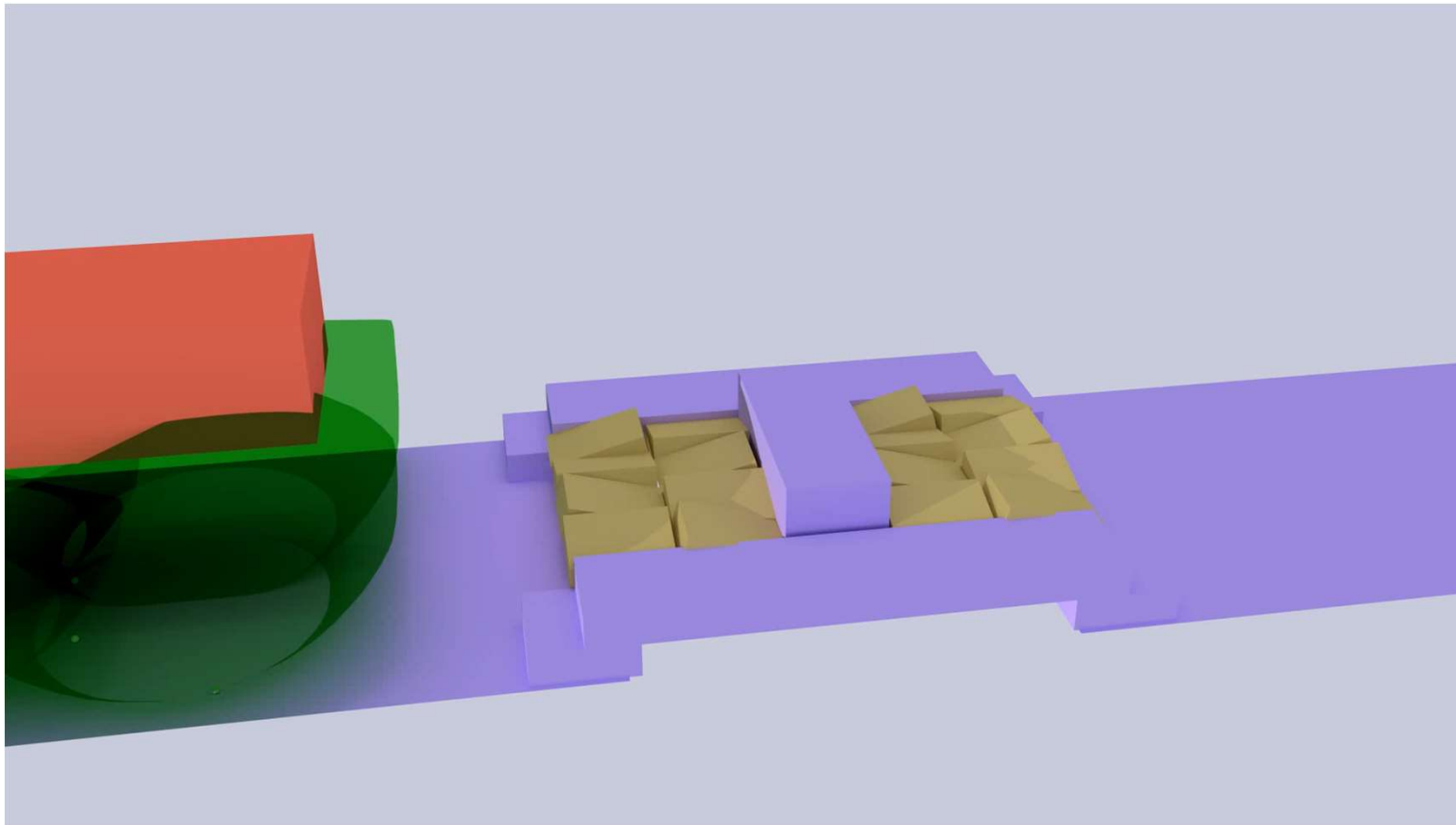
: Contact surfaces in **reachable workspace** of effector



Tonneau et al. 2015 (RSS) / 2019 (TRO)

# RRT planner to compute guide path in 6D

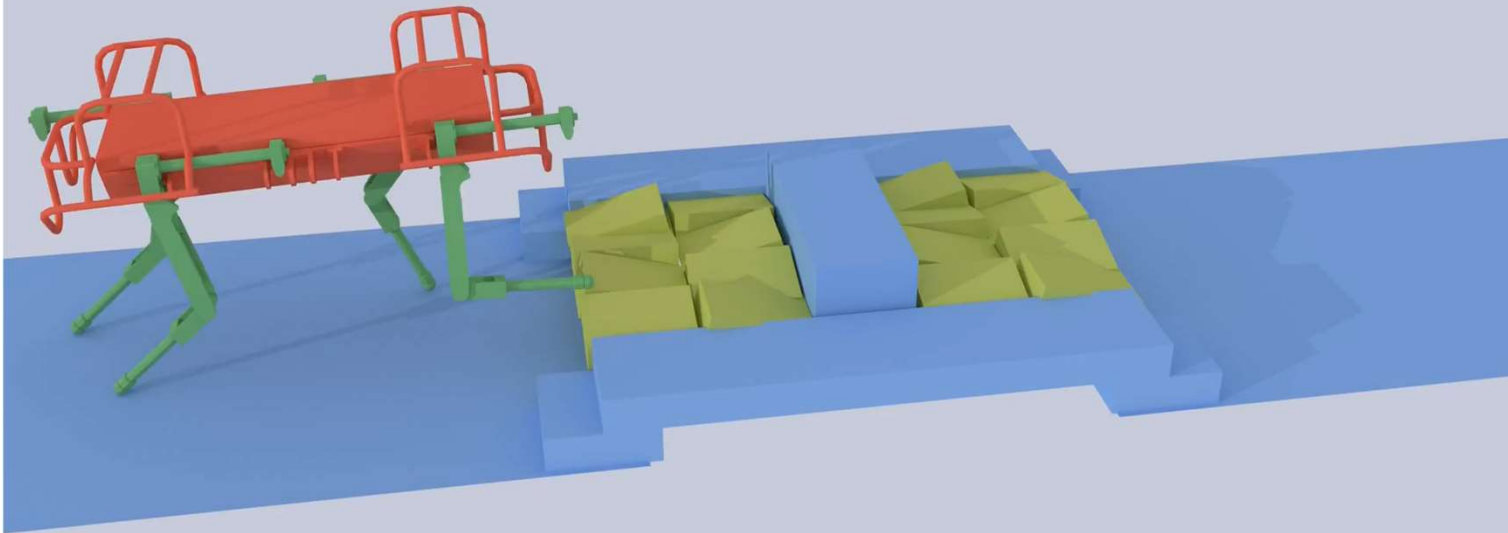
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# Random contact generator along guide (not SL1M)

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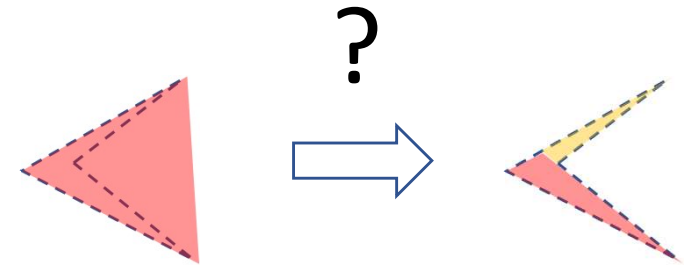
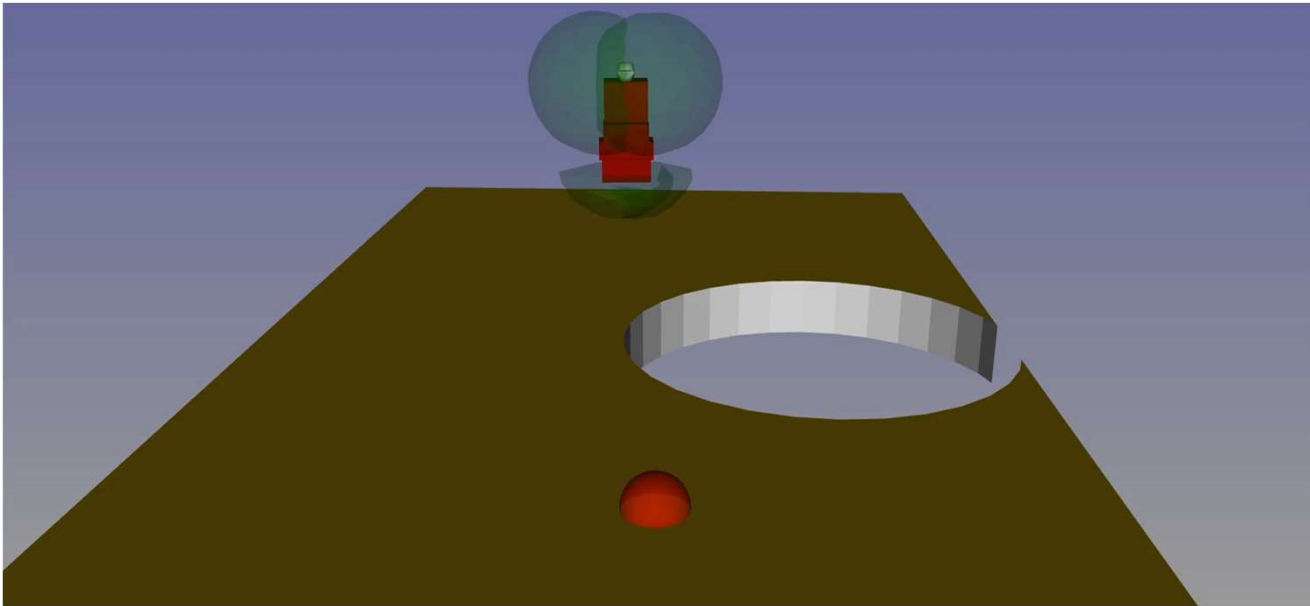
The guide path is then extended into a full body sequence using a dedicated generator



Complete computation time  $\approx 7s$

# Example of failure case

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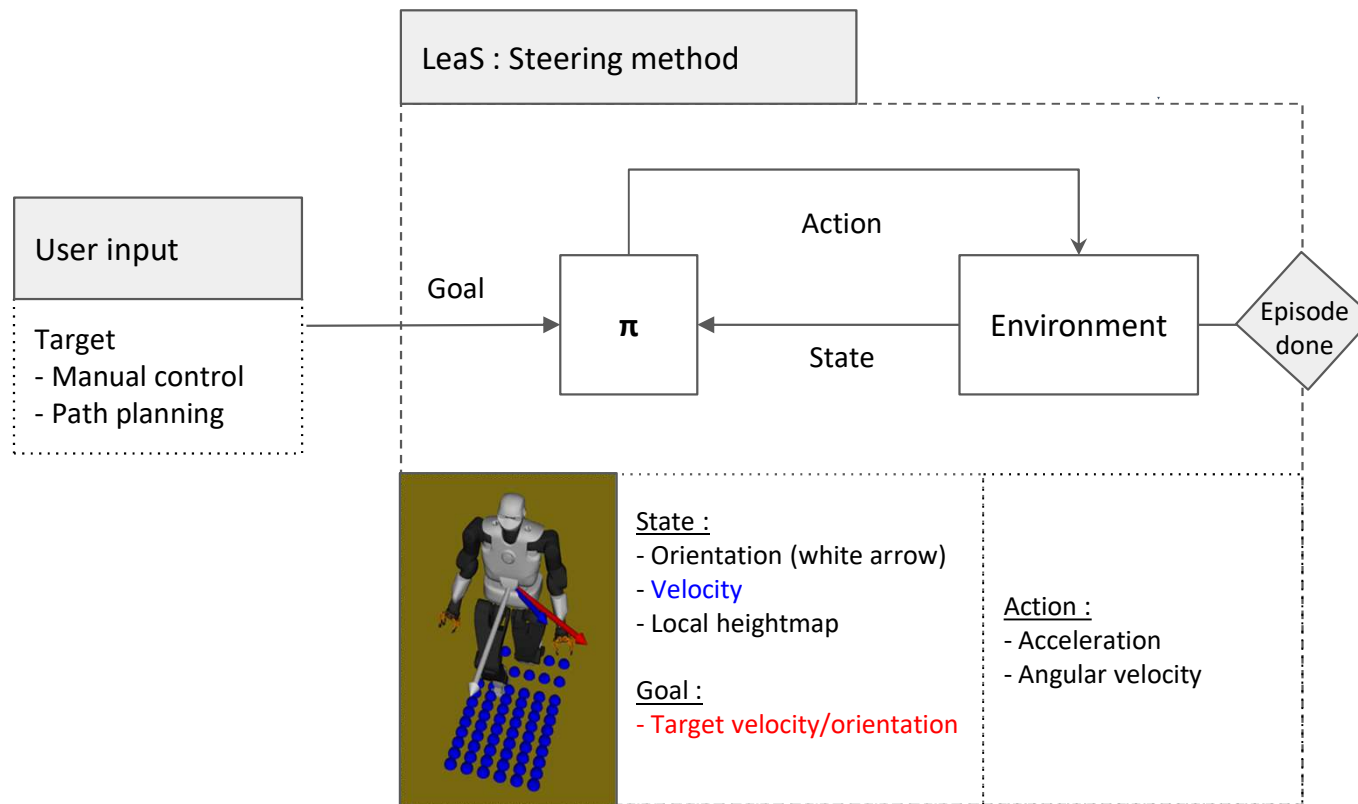
## The difficulty lies in the compromise between necessary and sufficient condition

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.Active research around the reachability condition since its original contribution, but more on P2 than on the actual condition

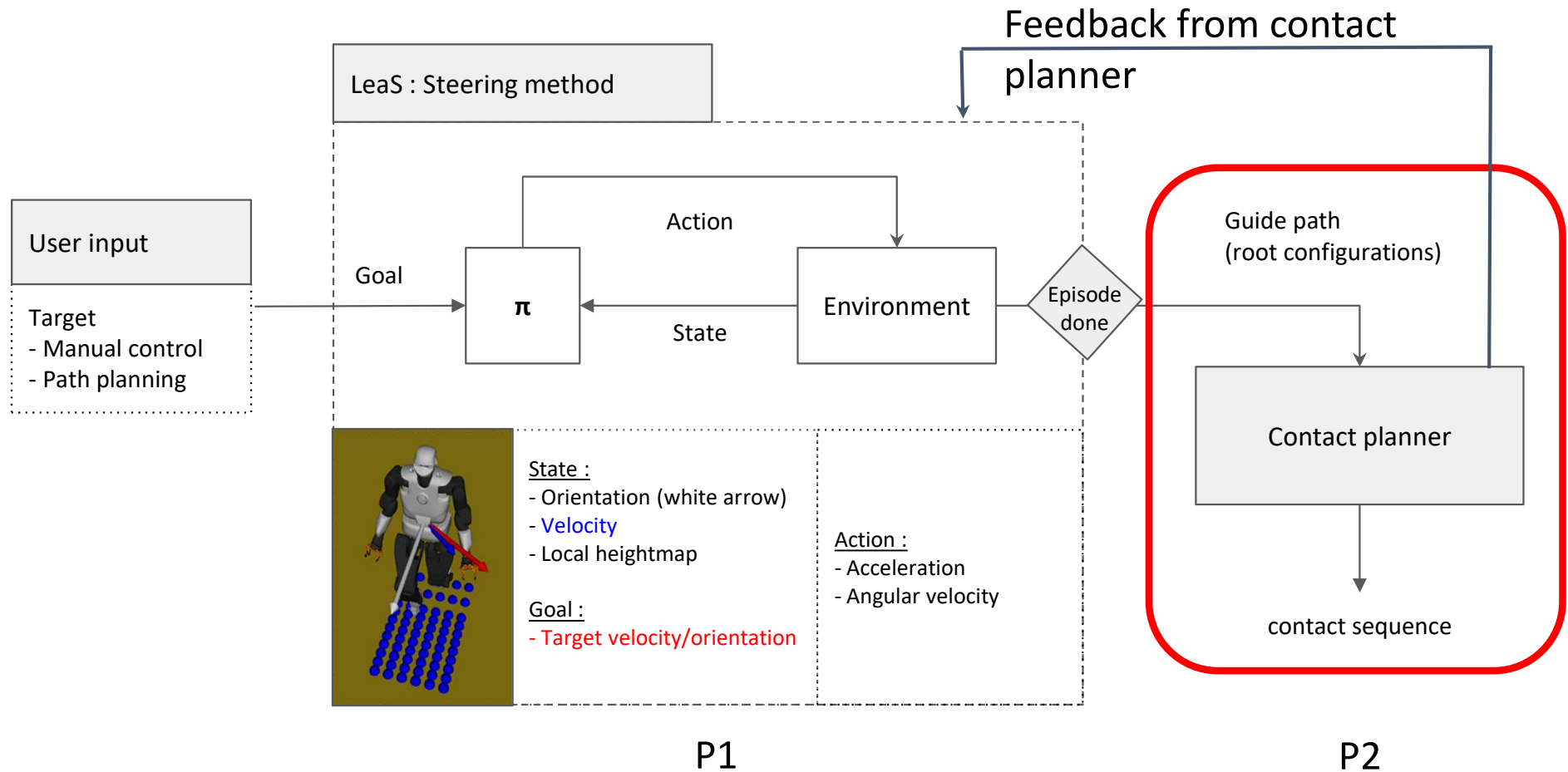
.Our proposition: Reinforcement Learning (RL) framework for improving the reachability condition

# Framework



P1

# Framework



# Training

- Arena generator : stairs, obstacles and bridges
- Random initial configuration and target velocity
- Asynchronous version of Proximal Policy optimization (PPO)<sup>1</sup> for 53 hours
- Sample-based contact planner<sup>2</sup>
- Talos model<sup>3</sup>



<sup>1</sup> "Proximal policy optimization algorithms" 2017

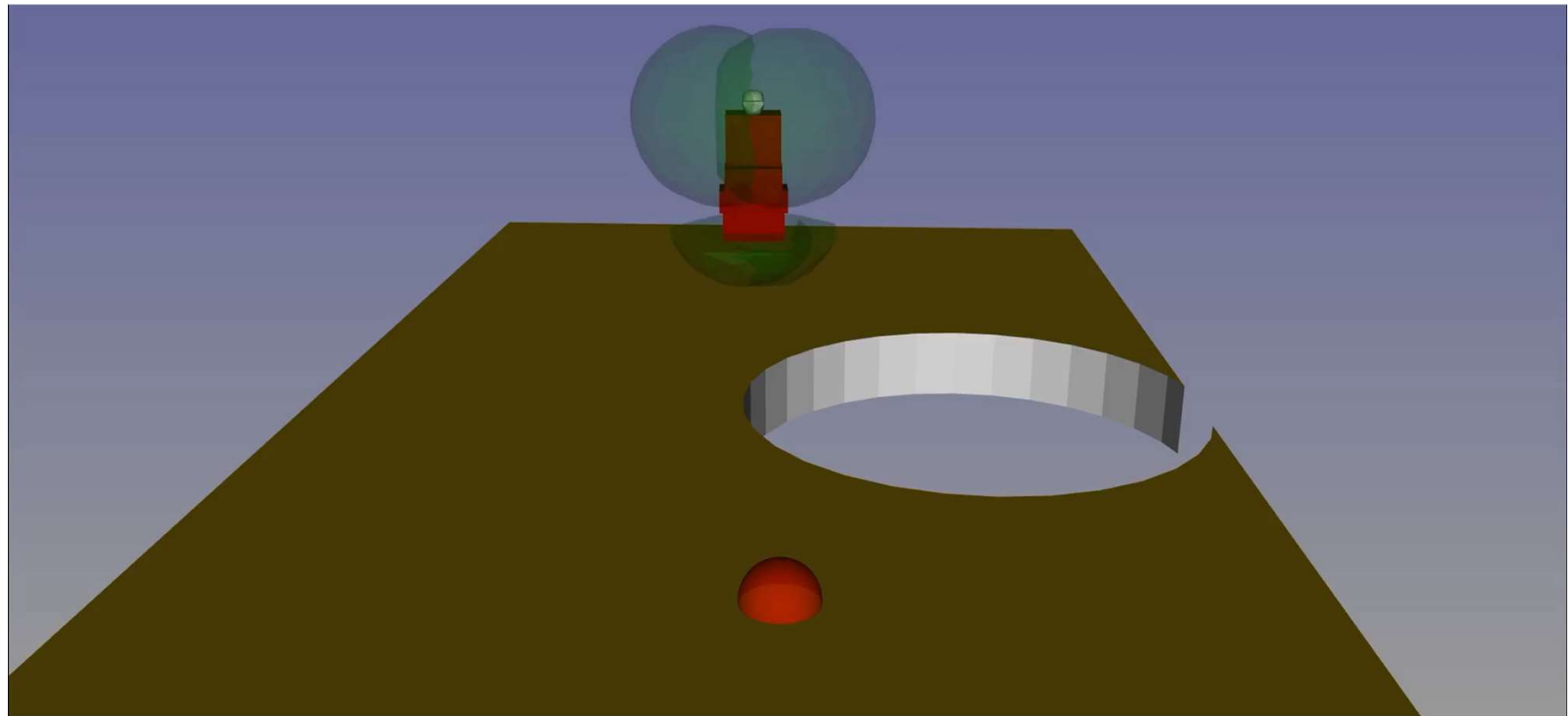
<sup>2</sup> "A Reachability-based planner for sequences of acyclic contacts in cluttered environments" 2015

<sup>3</sup> "TALOS: A new humanoid research platform targeted for industrial applications" 2017



Some results....

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# Early conclusions on RL for path planning

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.A robust approach:

- higher success rates, good generalisation
- Ongoing tests to quantify exactly to what extent

.Visual quality wrt pure sampling-based approach

.A limitation: no specification of precise target point

.Future work:

- SL1M as a contact planner to learn constraint tightening  
(Promising early results)

-Include continuous motion generation in feedback loop. Performance issue in terms of training ? Efficient models for motion generation ?